# Interleaved Mathematics Essential Connections 12



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## Contents

# Profitability of a Business Investigation



*When calculating potential profits from a business, it is important to consider the relative risks involved. In this investigation, you will compare the costs and profits of printing books through three different printers and make recommendations for which printer to use based on sales.*

## Brief

You will work in a group to investigate and compare three different business scenarios for a first-time author to print and sell books. You will use what you find out to create line graphs to show profitability over time for each scenario, including identifying the break-even point where the initial investment and costs result in profit. You will graph all three scenarios on the one set of axes, identify which is better initially, which is less risky, and which is better in the long run depending on sales.

## Skills Checklist

Skills you need to demonstrate for this assignment:

- Plot points on a Cartesian plane using tables of values
- Substitute values into expressions
- Interpret linear equations and graphs in practical situations
- Draw two or more functions on the same axes and interpret intersection points



## Scenarios

A first-time author gathered quotes from three printers to print her book. The book was 190 pages, to be printed in full-colour and perfect-bound. The author asked each printer to quote on printing 100, 200, 300 and 400 copies of her book. She expects to sell the book for $18 per copy but is not sure how many copies she will sell. Analyse and graph each scenario below, along with the sales price per book. Determine the equation for calculating the cost of each print run. Use the point of intersection to determine the break-even point. Graph all three print run quotes on the same axes and determine the intersections. Make recommendations for which print quote she should accept based on her sales expectations. If the author costs her time to produce the book at $1500, what is the minimum number of sales she needs?

| | 100 copies | 200 copies | 300 copies | 400 copies |
|---|---|---|---|---|
| Printer 1 | 1700 | 3000 | 4300 | 5600 |
| Printer 2 | 1850 | 2900 | 3950 | 5000 |
| Printer 3 | 2250 | 3100 | 3950 | 4800 |

## Hand in

- One line graph for each printer that shows the printing cost together with sales income. The break-even point and equation for the line should be identified.
- One graph with lines for each printer's quote, together with a fourth line to show the sales income from the books if sold at $18 per copy.
- A paragraph describing the risks and making recommendations for the author.

# 1. Relative size of numbers to 1 000 000

**Relative Size**

*Relative size underpins concepts such as fractions, percent, converting measurements and even probability. It is a fundamental idea that we will use to build many other concepts.*

## Experiment

Copy the number line beneath on a large piece of paper. Where would these numbers fit?
100, 1 000, 10 000, 100 000, 800 000

0 1 000 000

Does your first idea look right? Try adding in the half and quarter points consider if the spacing is reasonable. It is not unusual to try 10 different lines before the spacing works.
**This Number Line strategy will be used frequently in later lessons and chapters.**

## Explore

### Stimulus questions

1. What would 10 lots of 1 000 be?
   How about 100 lots of 1 000?
2. How many thousands make up 1 million?
3. What would half of 1 million be?
   Add it to your line.
4. What would a quarter of 1 million be?
   Add it to your line.

### Patterns and connections

How does knowledge of our base-ten system for place value help you to figure out the positions of numbers on the number line?

## Evaluate

What key numbers or position points can be used to help you figure out positions on a number line from 0 to 1 000 000?
If you changed your mind, what did you change your mind about?

## Explain

There is only one correct position for each number.

- How do you know that your numbers are in the right place?
- What numbers or positions would you start with next time?

## Extend

1. Builders work in millimetres. What measurements would be equal to 1 million millimetres?
2. If a plan showed a measurement of 10 000 mm, how long would that be in metres or kilometres?
3. Our place value system allows us to write numbers as powers of ten. For example, 10 is the same as $10^1$. 100 is the same as $10^2$. Use this information to express 1 million in powers of ten.
4. Draw a 1m long number line (E.g., with chalk on the floor). On the left-hand end, write 0 and on the right-hand end write $10^6$. Add $10^1$, $10^2$, $10^3$, $10^4$ and $10^5$ to your line.

# 2. Relative size of fractions, decimals and percentage

**RELATIVE SIZE** **PROPORTIONAL REASONING**

*Fractions, decimal numbers and percentages are different ways of representing the same amount. Previously, we used the strategy* ***Of A Dollar*** *to help us turn fractions into decimals without requiring a formal process. In this task we will review the* ***Of a Dollar*** *strategy and construct a number line to explore the relative size of fractions, decimals and percentages.*

## Review

One particular operation will help you to convert fractions into decimals and therefore, into percentages. Use a calculator to determine which operation will correctly fill the blank space in the equation below, given that we know that ½ a dollar is 50c or 0.5.

$$1 \_\_\_ 2 = 0.5$$

What operation can you use to make this work?
This is our new strategy: **Fractions are** ___________

## Experiment

Convert the following amounts into fractions, decimal numbers and percentages, then place them on a **Number Line** that stretches between 0 and 1:

$\frac{2}{5}$ 30% $\frac{3}{4}$ 0.65 $\frac{2}{3}$ 0.8 $\frac{3}{20}$ 50% $\frac{17}{20}$

0 1

> **RELATIVE SIZE**
> **PROPORTIONAL REASONING**
> **Of a Dollar Strategy**
> Decimal numbers are "base ten". Money is also "base ten", so we can use it to understand converting fractions to decimals and percentages.
>
> What is one quarter **Of A Dollar**?
> How many cents are there?
> That is ¼ as a percentage.
> Write the amount in dollars and cents.
> That is how we write ¼ as a decimal number.
> ¼ = 25c = 25% = 0.25



## Explore and Evaluate

Does your first idea look right? Try adding in all the other tenths and consider if the spacing is reasonable.

- Where would one half fit?
- Where would each of the quarters fit?
- What other decimal numbers and fractions do you know that might be helpful?

## Explain and Extend

What key numbers or position points can be used to help you figure out positions on a number line from 0 to 1?
Write a simple explanation for steps to use to convert any fraction into a decimal number or percentage, assuming you have a calculator.
Use the digits 3, 4, 6, 7 and 8 to create three fractions to add to your number line.

# 3. Relative size and unit conversion (length, mass, capacity)

## RELATIVE SIZE

*In the Year 11 program, we used a* **Number Line** *and a* **Place Value Chart** *to show links between units of length, mass and capacity. In this lesson, we will review what we learned and apply Relative Size to convert units of measurement.*

## Review

Use a measuring tape or ruler to measure the lengths of the following objects. Record the lengths in centimetres, millimetres and metres. Record any patterns you can find.

| Object | Metres | Centimetres | Millimetres | What patterns can you see between the measurements? |
|---|---|---|---|---|
| Shoe length | | | | |
| Board length | | | | |
| Arm length | | | | |

## Explore and Explain

The **Place Value Chart** below uses the pattern that you have found above to convert easily between different units of length. The example given shows converting a measurement of 68cm. Note that the digits of 6 and 8 always stay in the same position, but the decimal point changes to the unit indicated in the left hand column. Explore how the chart works, then use the blank rows in the column to convert your measurement for the length of your shoe.

| | Thousands (of ones) | Hundreds (of ones) | Tens (of ones) | Ones. | Tenths (of ones) | Hundredths (of ones) | Thousandths (of ones) |
|---|---|---|---|---|---|---|---|
| | **kilo** kilometres | | | **Base** metres | | **centi** centimetres | **milli** millimetres |
| cm | | | | | 6 | 8. | |
| m | | | | 0. | 6 | 8 | |
| mm | | | | | 6 | 8 | 0. |
| km | 0. | 0 | 0 | 0 | 6 | 8 | |
| Shoe cm | | | | | | | |
| m | | | | | | | |
| mm | | | | | | | |
| km | | | | | | | |

Explain how to use the strategy to convert between units of length. How could you use a calculator to convert between units of length by multiplying or dividing?

## Apply

The same thinking that you used to convert units of length also applies to converting measurements of mass and capacity. Practise your skills for length conversion in Set A below, then use this thinking to complete Set B (Capacity) and Set C (mass).

### Set A: Convert units of length

Sketch a **Place Value Chart** on your book and use it to convert the following lengths.

1. 8cm into m
2. 8mm into cm
3. 8m into km
4. 8km into m
5. 80cm into km
6. 80mm into m
7. 0.8m into mm
8. 0.8cm into km
9. 0.8m into km

### Capacity in the **Place Value Chart**

The **Place Value Chart** below shows 2L converted into mL and kL.

| | Thousands | Hundreds | Tens | Ones. | Tenths | Hundredths | Thousandths |
|---|---|---|---|---|---|---|---|
| | kilolitres | | | litres | | | millilitres |
| L | | | | 2. | | | |
| mL | | | | 2 | 0 | 0 | 0. |
| kL | 0. | 0 | 0 | 2 | | | |

### Set B: Convert units of capacity

Sketch a **Place Value Chart** on your book and use it to convert the following capacities.

10. 8L into mL
11. 8mL into L
12. 8kL into L
13. 8L into kL
14. 8mL into kL
15. 8kL into mL
16. 0.8L into mL
17. 0.8mL into L
18. 0.8L into kL

### Set C: Convert units of mass

Sketch a **Place Value Chart** on your book and use it to convert the following masses.

19. 8mg into g
20. 8g into mg
21. 8g into kg
22. 8mg into kg
23. 8kg into g
24. 8kg into mg
25. 0.8kg into g
26. 0.8mg into g
27. 0.8g into kg

## Connect and Generalise

We can use multiplication and division to easily convert measurement units. To do this, we need to know how many of the original units make up the converted units. For example, to convert between cm and km, we know that 100 cm make up one metre and 1000m make up one kilometre, therefore 100 000 cm make up one km.

- To convert between cm and km, we would divide by 100 000.
- To convert between km and cm we would multiply by 100 000.

How would we convert between:

- m and mm and vice versa?
- mL and L and vice versa?
- kg and mg and vice versa?

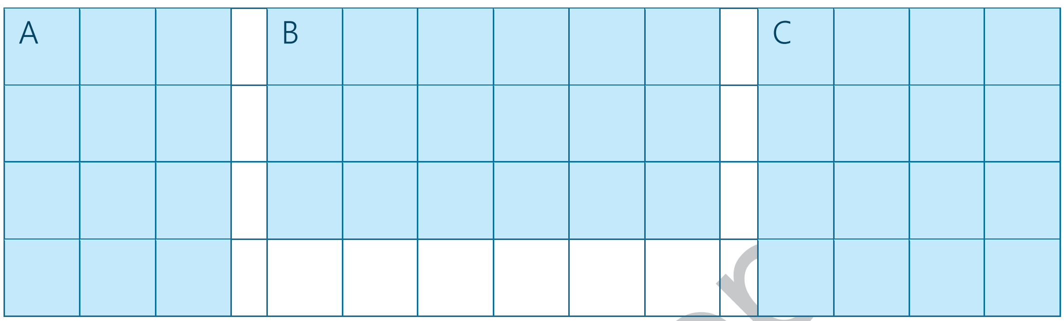
# 4. Review arrays and area

**MULTIPLICATIVE THINKING**

*Area is one of the most common calculations used in building, farming, renovating and town planning. In this lesson, you will review using the* ***Array Strategy*** *to work out area of rectangles.*

## Review

How many square centimetres make up the area of each rectangle? Show your working.




How could you use the base measurement and the height measurement from each rectangle to determine its area? Write a formula using the variables A (area), B (base length) and H (perpendicular height).

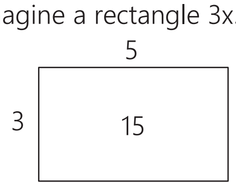
## Explore and Explain

Calculate the area of the following rectangles:

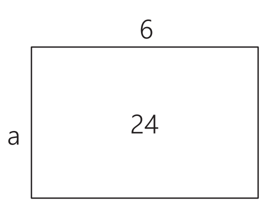
1. A rectangle that was twice as long as it was high. The height was 5cm.
2. A square that was half a metre long – how many square metres does it take up altogether?
3. A rectangle that was 1.5m long and 60cm high.

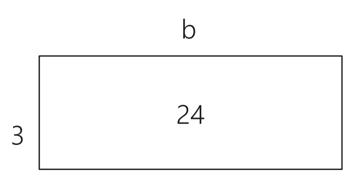
## Apply Array Strategy to link Division and Fractions

Imagine a rectangle 3x5. Read the information in the box.




Consider the relationship between multiplication, division and fractions. Use this to work out the missing side lengths on the following rectangles.




**MULTIPLICATIVE THINKING**
**PROPORTIONAL REASONING**

The diagram shows that
3 x 5 = 15
This also means that
15 ÷ 3 = 5 and
15 ÷ 5 = 3.
Using our **Fractions are Division** strategy, we can also write:

- $\frac{15}{5} = 3$ and $3 = \frac{15}{5}$
- $\frac{15}{3} = 5$ and $5 = \frac{15}{3}$

# 5. Review ratios and scales

MULTIPLICATIVE THINKING  PROPORTIONAL REASONING

*Ratios are one of the most commonly used mathematical tools in the real world. In this lesson, you will review what you have previously learned about ratios and using a* **Relationship Table** *to help with calculations.*

## Review

**Ratios** focus on the parts **in relation to each other**, in their simplest form. Use the thinking in the worked example to express the following ratios.

1. In a class of students, 2 children played cricket for every 3 children who played soccer.
   a. Write this as a ratio.
   b. If there were 25 children in the class in total, how many played soccer and how many played cricket? How did you work it out?
2. A bag of lollies had a ratio of red: yellow: blue of 4:3:2
   a. If there were 9 yellow lollies, how many red and blue were there? Explain why.
   b. If there were 36 lollies altogether, how many would there be of each colour?

**Worked example**

To cook most types of rice we use a ratio to determine how much water to add. For every **2** cups of rice, we add **3** cups of water. We use **5** cups of ingredients in total.

The **ratio** is 2:3

The **fraction** of rice is $\frac{2}{5}$

The **fraction** of water is $\frac{3}{5}$

The answer to 2b (above) is 16 red, 12 yellow and 8 blue. Use this answer to complete the **Relationship Table** below and remember how it works.

| | Red | Yellow | Blue | Total |
|---|---|---|---|---|
| Original ratio | 4 | 3 | 2 | 9 |
| Converted ratio | | | | 36 |

How did we change the 9 into a 36? Apply that same change to all other numbers to see what happens.

## Explain

Write a simple process for dividing a quantity into a ratio using the Relationship Table.

## Apply

Scales on maps and plans use ratios to show what 1cm on the drawing represents in real life. For example, a ratio of 1:10 would mean 1cm on the drawing was 10cm in real life.

1. Commonly, house plans are drawn using a scale of either 1:100 or 1:50. What distance would 1cm on a house plan represent in real life using each scale?
2. A bedroom on a house plan measured 3.3cm x 3.0cm. What scale would likely have been used and what would the area of the bedroom be? What would its perimeter be?
3. A surveyor was drawing a plan of a playground using a scale of 1:100. It was 13500mm long and 8900mm wide in real life. What was the area of the playground in $cm^2$ on the plan?

# 6. Review fractions as division and apply to equations

**MULTIPLICATIVE THINKING** **PROPORTIONAL REASONING**

*Previously, we used the strategy* ***Fractions are Division*** *to help us turn fractions into decimals and to determine the side lengths in rectangles. In this task, we will use this strategy to help us to rearrange equations so that we can calculate the value of an unknown.*

## Explore and Explain

When writing equations with unknowns, we usually use fractions instead of writing division.

For example, $\frac{a}{4} = 5$ means that $a \div 4 = 5$.

Use this to work out what "a" would be by rearranging the equation. Explain how you did it.

> **RELATIVE SIZE**
> **PROPORTIONAL REASONING**
> In Lessons 2 and 4 we learned the strategy: **Fractions are Division**.
>
> Fractions and division are different representations of the same thing. The vinculum in the fractions symbol means divide, so
> - $\frac{3}{5} = 3 \div 5$
> - $\frac{5}{6} = 5 \div 6$

## Apply and Practise

Apply the same process to rearrange each of the following equations. Use a new line for each step in your working. For each equation, calculate the value of the pronumeral "a".

1. $\frac{a}{4} = 5$
2. $\frac{a}{2} = 5$
3. $\frac{2a}{4} = 5$
4. $\frac{(a - 2)}{4} = 5$
5. $\frac{a}{4} = \frac{1}{2}$



One idea that makes equations simpler to handle is to draw a box around the part of the equation with the unknown, so that you can use the **Post-It Note Algebra** strategy. Check out the Worked Example and deduce how it works. Work out what goes in the box first, then deal with what is in the box next. Try this on the next set of questions.

6. $\frac{20}{a} = 5$
7. $\frac{20}{2a} = 5$
8. $\frac{10}{(a + 2)} = 2$
9. $\frac{24}{a} = 4$
10. $\frac{24}{2a} = 6$

> **Worked example**
>
> $$\frac{(a - 2)}{4} = 5$$
>
> $$\frac{\blacksquare}{4} = 5$$
>
> $$a - 2 = 20$$

## Evaluate

Which of the equations above did you find the most difficult to calculate? Why do you think they were harder?

# 7. Change the subject of equations

GENERALISING  MULTIPLICATIVE THINKING

*Most jobs require the use of some kind of formula for calculations. It is important to be able to rearrange a formula so that you can find the information you need. In this lesson, you will apply the* **Post-It-Note Algebra** *and* **Fractions are Division** *strategies to help you to change the subject of an equation or formula by rearranging it.*

## Experiment and Explore

Read through the information on inverse operations in the box. This will be useful for you to remember when you are trying to rearrange the equations below. Try rearranging the equation or formula in each of the scenarios below to answer the question.

> MULTIPLICATIVE THINKING
> PROPORTIONAL REASONING
>
> In previous lessons we learned about inverse operations.
>
> - Multiplication is the inverse of Division.
> - Addition is the inverse of Subtraction.
>
> We are going to use these relationships, along with the strategy **Fractions are Division**, to figure out how to rearrange equations and calculate the value of an unknown.

### Scenario 1: Tax and take-home pay

My gross income (g) includes the tax (t) that I pay. My net income (n) is my "after tax" income.

We can represent this using the Partition It strategy:

| | |
|---|---|
| gross income = | |
| net income + | tax |

We could write this as:

**$g = n + t$** or **$n + t = g$**

1. Rearrange the equation to show how to calculate n.
2. Tax (t) is calculated at $\frac{1}{5}$ of my gross income (g). Write an equation for calculating t.
3. Substitute your new equation for tax into the formula instead of using t.
   Your formula should now only use g and n as variables.
4. If my gross income is \$100, what is my net income?
5. If my net income is \$100, what is my gross income?

### Scenario 2: Speed of travel

Speed (s) is calculated by dividing the distance travelled (d) by the time the journey took (t).

6. Use an array strategy to determine where to write distance (d) and time (t) on this rectangle to show the multiplicative relationships.
7. Write the formula for calculating speed.
8. Rearrange the formula so that the time is the subject.
9. If I travelled at a speed of 100km/h for a distance of 250km, how long did it take me?
10. Rearrange the formula so that distance is the subject.
11. I drove at a speed of 80km/h for 3 hours. How far did I travel?

s

## Apply and Practise

### Scenario 3: Income from sales

A shop sold shirts and hats. Each shirt (s) was worth \$10. Each hat (h) was worth \$15.

12. Write an equation to show how to calculate the total income (t) for the day using the number of shirts (s) and number of hats (h) sold.
13. One day, the shop sold 5 shirts and 6 hats. How much income did the shop make?
14. Rearrange the equation to calculate how many shirts were sold using the total income and number of hats sold.
15. One day, the shop had an income of \$90. They sold 4 hats. How many shirts were sold?
16. Rearrange the equations to calculate how many hats were sold using the total income and number of shirts sold.
17. One day, the shop sold 5 shirts and some hats. The total income was \$80. How many hats were sold?

### Scenario 4: Booking a conference dinner

Organisers for a conference wanted to book a dinner at a local restaurant. The restaurant charged a booking fee (b) as well as a food cost per person (f). The total cost for the conference organisers (t) could be calculated as the number of attendees (n) multiplied by the food cost per person (f), plus the booking fee (b).

18. Write an equation to show the total cost (t) using variables.
19. What would the total cost be if the booking fee was \$50, the number of attendees was 20 and the food cost per person was \$15?
20. What would the booking fee be if the total cost was \$200, 10 people went, and the food cost per person was \$12?

Complete the following **Relationship Table**, calculating the total cost when different numbers of people attend. The food cost per person is set at \$10. The booking fee is set at \$50.

| Number of attendees | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Cost | | | | | | |

### Graph the result: You will need this for the following few lessons.

Draw a graph of the values in the table.

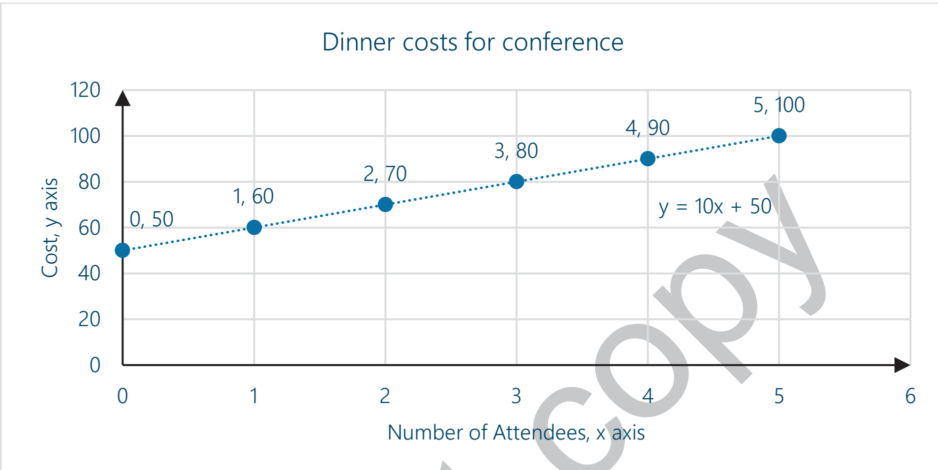
- Write a scale to show the number of attendees on the x-axis (horizontal axis), and a scale to show the total cost on the y-axis (vertical axis) E.g., on the x-axis you should have 0-5.
- To create a point on your graph: line up the number of attendees (x-axis coordinate) and the total cost (y-axis coordinate) and draw a point or dot where they meet. You should end up with 6 points on your graph.
- What do you notice about the points on your graph?

# 8. Linear equations and tables of values

GENERALISING MULTIPLICATIVE THINKING

*In Lesson 7, Scenario 4, we used a formula to complete a table of values. By plotting the values on the Cartesian Plane, we could see that they formed a line. This is called a linear function.*

- *The graph below shows: cost = food x number of attendees + booking fee, or cost = fn + b.*
- *The food cost was set at $10 per attendee. The booking fee was $50. So, cost = 10n + 50.*
- *We plotted the n on the x-axis and c on the y-axis, so the graph below shows:* **y = 10x + 50**






## Explore

### Let's explore the graph above:

1. At what point does the graph cross the y axis? This is called the **intercept**. Where is that in the equation y = 10x + 50?
2. Each time we add one more attendee, the total cost goes up by the same amount. This is called the **slope** or **gradient**. Where is that in the equation y = 10x + 50?

In the questions that follow, we are going to vary the scenario and see what happens. For each question below you need to:

- Write the equation to calculate the total cost.
- Create a table of values just like in Lesson 7 (use the **Relationship Table** strategy).
- Draw a new set of points on your existing graph and join them to show a line.
- Identify the intercept and the slope (or gradient) in the equation.

## Apply and Practise

Complete the steps above for the following scenarios.

3. The booking fee changes to $20, but the food cost per attendee stays at $10.
4. The booking fee changes to $60, but the food cost per attendee stays at $10.
5. The booking fee changes to $10, but the food cost per attendee stays at $10.
6. What do you notice about each of your lines?

# 9. Plotting points from tables of values

GENERALISING MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*In the previous two lessons, you have used an equation to create a table of values, then plot it on the Cartesian Plane as a graph. In this lesson, you will work backwards from a table of values to create a graph, then think about the equation that might go with it.*

## Experiment and Explore

Examine the table below. It uses the same calculation for a conference dinner that you explored in lessons 7-8. What can you see that has changed? What has stayed the same?

| Number of attendees (x) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Cost (y) | 30 | 40 | 50 | 60 | 70 | 80 |

1. Graph the points on the same axes that you have used for the previous lessons. What do you notice about the new line? How is it similar to the other lines?
2. What is the intercept for your line? Where does the graph meet the y-axis?
3. What is the slope or gradient for your graph (remember: this is the amount that total cost goes up by for each additional person)?

## Explain

4. Try to use the slope and the intercept to write an equation for your line. It should be similar to the equations for lines in Lesson 8. Discuss your ideas with someone else. Explain how to use the slope and intercept to write the equation for the line.

## Apply and Practise

The scenarios below all vary the costs for the conference dinner. Complete the 4 steps above for each of the tables of values. Graph the points, identify the intercept and slope (gradient), then write the equation. Watch out though - this time the food cost changes!

### Scenario A

| Number of attendees (x) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Cost (y) | 30 | 50 | 70 | 90 | 110 | 130 |

### Scenario B

| Number of attendees (x) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Cost (y) | 30 | 35 | 40 | 45 | 50 | 55 |

# 10. Plotting points from tables of values 2

GENERALISING MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*In the previous lessons, you have graphed equations for conference dinners. In this lesson, you will extend that same thinking to other scenarios, use tables of values to create graphs, then think about how to calculate the intercept and the slope (gradient) for each graph.*

## Extend

Examine the table below. It shows what happened when a litre of water was heated on a stove for 5 minutes. The temperature was taken every minute.

| Time (minutes) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Temperature (degrees) | 25 | 35 | 45 | 55 | 65 | 75 |

1. Graph the points on a new set of axes, with time on the x-axis and temperature on the y-axis. What do you notice about the line? How is it similar to your previous graphs?
2. What is the intercept for your line? Where does the graph meet the y-axis?
3. What is the slope (gradient) for your graph (remember: this is the amount the total cost goes up by for each additional person)?
4. Use the slope and the intercept to write an equation for your line, starting with y=.

## Apply and Practise

The scenarios below all create linear graphs. For each scenario, graph the points, identify the intercept and slope (gradient), then try writing the equation.

### Scenario A: Excursion costs are calculated using the bus fee and a cost per student

| Number of students (x) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Cost (y) | 50 | 55 | 60 | 65 | 70 | 75 |

### Scenario B: Printing costs are calculated using the set-up fee and a cost per book

| Number of books (x) | 0 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| Total Cost (y) | 200 | 300 | 400 | 500 | 600 | 700 |

*Note: the number of books is going up in 10s not 1s.

### Scenario C: I am saving money for a holiday. I have some saved and earn some each week.

| Number of weeks (x) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Total Savings (y) | 60 | 100 | 140 | 180 | 220 | 260 |

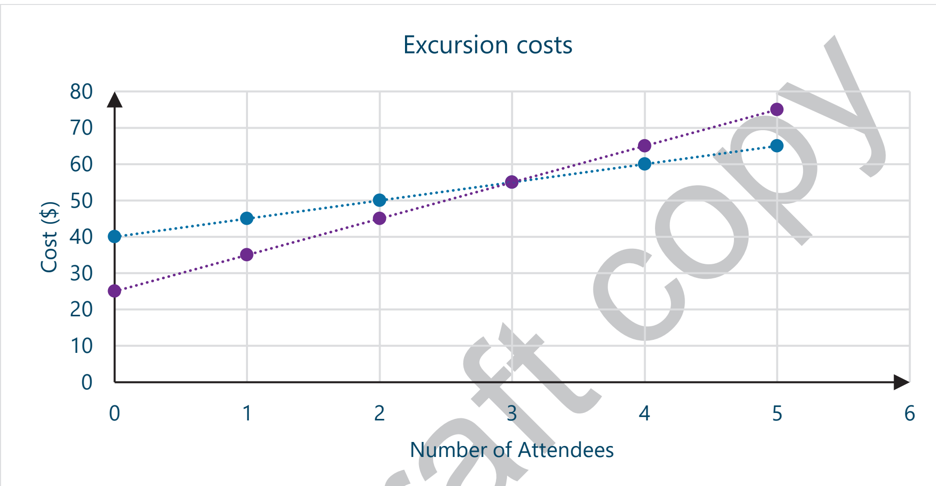
# 11. Interpret linear equations

GENERALISING MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*In lessons 7-10, you created graphs using points from a table of values or from equations. In this lesson, you will learn how to apply what you have learned to interpret linear equations and think about the slope (gradient) and intercept.*

## Experiment

Examine the graph below. It shows two scenarios for costing of an excursion. Each has a booking fee and a cost per attendee. Use the information to create a table of values for each scenario. Scenario A has the point (0,40). Scenario B has the point (0,25).






1. Create a table of values for each scenario.
2. For Scenario A, what is the intercept? For Scenario A, what is the slope (or gradient)?
3. For Scenario A, we can write an equation in slope-intercept format.
   This would be written as: **y = 5x + 40**. How do you think the equation was created?
4. For Scenario B, what is the intercept? For Scenario B, what is the slope?
5. Write an equation for Scenario B in slope-intercept format.

## Explore

The following linear equations are written in slope-intercept format. Use the equations to create a table of values for each equation, then sketch a graph of each line on the same set of axes. Write the equation next to each line that you draw. What do you notice?

### Set A: altering the intercept

6. $y = x$
7. $y = x + 1$
8. $y = x + 2$

### Set B: altering the gradient

9. $y = 2x$
10. $y = 3x$
11. $y = \frac{1}{2}x$

# 12. Calculating gradient for linear equations

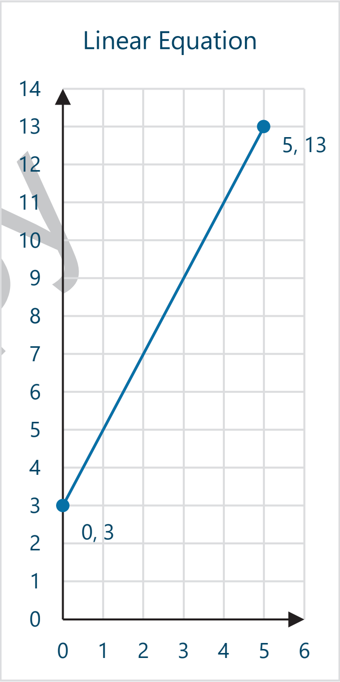
GENERALISING MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*The slope-intercept format enables us to easily compare and sketch equations. In this lesson, you will apply the thinking you have used over the previous lessons to calculate the gradient for any linear equation using two points on a graph. As you have noticed in previous lessons, the bigger the gradient, the steeper the slope of the graph.*

## Explore

This graph shows a linear equation, with only 2 points.

1. Calculating the **intercept** from this graph is relatively easy as the point where it crosses the axis is shown. What is it?
2. To visually work out the slope (gradient), we can think about how much the y-coordinate goes up by, when the x-coordinate goes up by 1. For this graph, it is relatively easy to see what happens. What is the slope (gradient)?




When the gradient (slope) is not easy to see visually, we can use two points and apply our **Fractions are Division** strategy.

- We label our two points $(x_1,y_1)$ and $(x_2,y_2)$.
- We calculate the **rise** between the two points by subtracting the y-values $(y_2\text{-}y_1)$.
- We calculate the **run** between the two points by subtracting the x-values $(x_2\text{-}x_1)$.
- We use the rise and run to calculate **gradient**.

  Gradient $= \frac{\text{rise}}{\text{run}}$

3. Use the two points given to calculate the rise.
4. Use the two points given to calculate the run.
5. Use the rise and run to calculate the gradient, using the **Fractions are Division** strategy. Check: is your answer the same as you obtained visually? If not, check with your teacher.

## Apply and Practise

For each of the set of points below: plot them on an axis, label the points, calculate the intercept, calculate the gradient using rise/run, and write the equation for the line.

6. (0,2) and (4,6)
7. (0,2) and (4,10)
8. (0,2) and (4,4)
9. (0,1) and (4,5)
10. (0,1) and (4,13)
11. (0,-1) and (4,3)

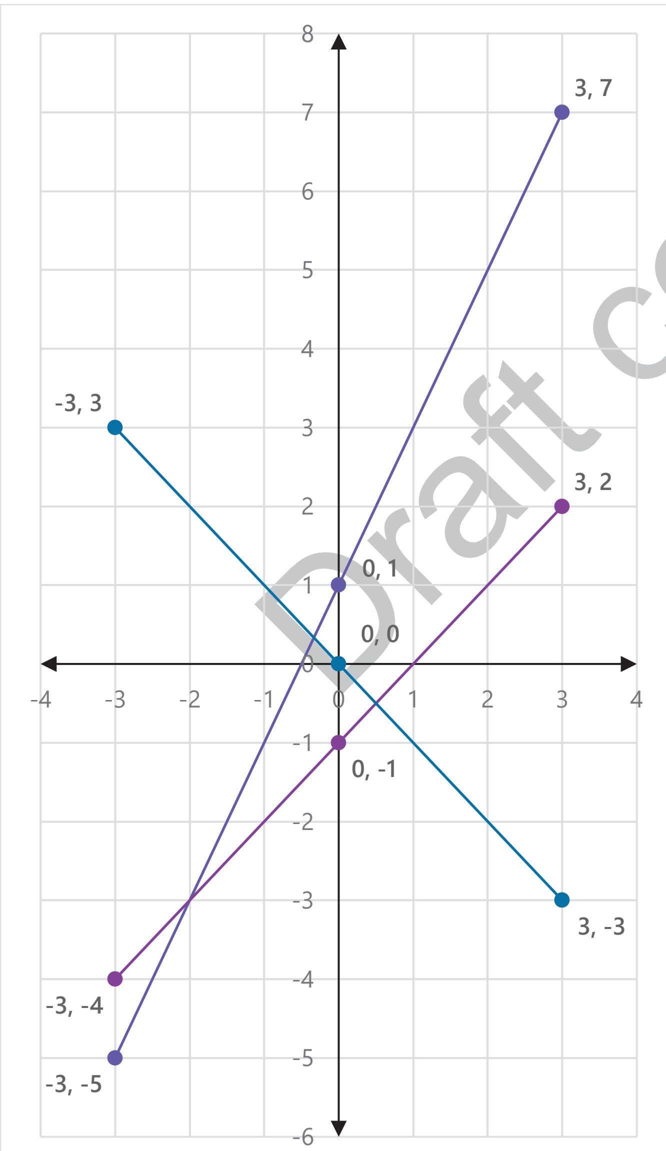
## 13. Graphing negative values

GENERALISING · MULTIPLICATIVE THINKING · PROPORTIONAL REASONING

*In the last question of the previous lesson, you ended up with a point on your graph that had a negative value. Negative values are important to show losses and expenditure. Use the graph below to figure out how linear equations work across all four quadrants.*

### Experiment

Examine the graph below. Create a table of values for each of the lines shown.
Calculate the intercept and gradient for each line. Write the equation next to each line.




### Explore

Notice how the gradient of the teal line is very different to the others? Describe how it is different and why.

Consider the differences between lines with a:

- **positive gradient** (rising, left to right)
- **negative gradient** (falling, left to right).

The teal line has a gradient of -1. Sketch a line that would have a gradient of -2.

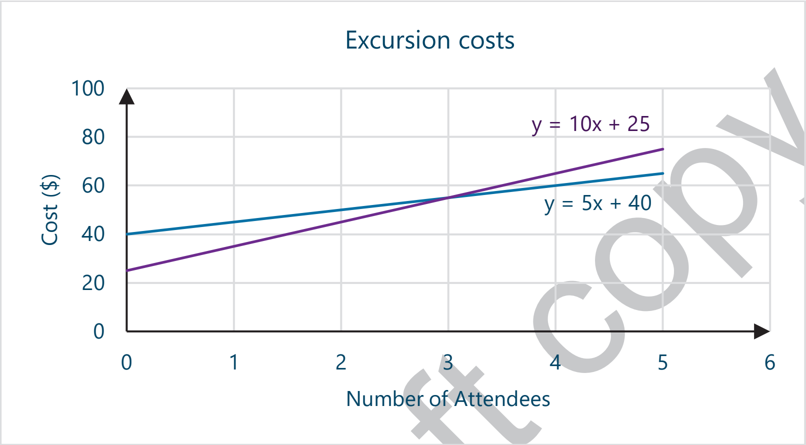
# 14. Intersection points

GENERALISING  MULTIPLICATIVE THINKING  PROPORTIONAL REASONING

*By graphing data, we can easily compare costs and profits for different scenarios and work out which is better. In this lesson, you will calculate the intersection point for two linear equations.*

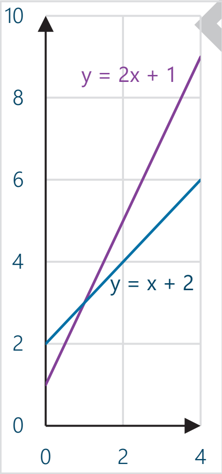
## Explore

The scenario shown in the graph below was used in Lesson 11. It shows two scenarios for costing of an excursion. Which scenario is best in which circumstances? At which point does it change? The point at which the two graphs intersect is called the intersection point.






On the graph above, you can fairly easily see that the intersection point is (3,55). Often, this is not quite so easy, so it requires some logical thinking to work through.




Firstly, we need to notice that at the intersection point both the x-coordinate and the y-coordinate are equal. That means that the equations will give the same y-coordinate output at that point.
At the intersection point, the two equations are equal:

$$2x + 1 = x + 2$$

1. Rearrange the equation so that the x is on the same side and the numbers are on the other. What did you end up with? You should have a value for x.
2. Now that we know the x-coordinate, we can substitute x into either of the equations to find the y-coordinate. What it is?
   Check using the second equation. You should get the same y value.

Use what you have learned to calculate the intersection points for the graphs in your investigation.

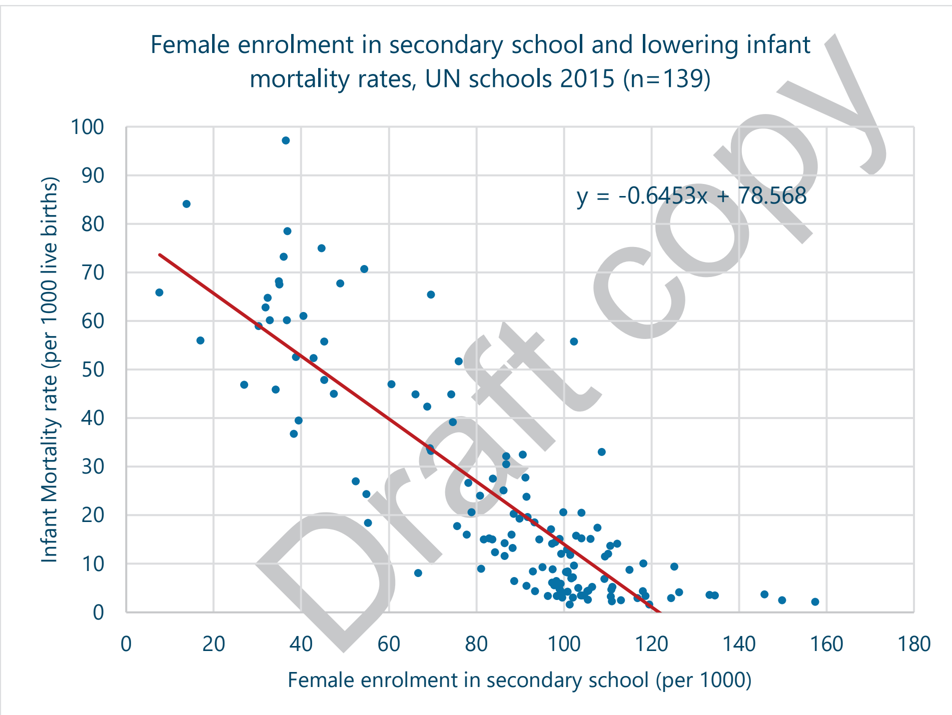
# 15. Real world data and straight line equations

GENERALISING   MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

*One of the very important uses for straight line equations in the real world is using them to describe trends in data. The graph below is a scatterplot which relates two pieces of information.*

## Evaluate

The graph below shows 139 data points, each representing a different country. Each data point plots the rate of rate of female enrolment in secondary school with the rate of infant mortality. A line of best fit, or trend line, has been added to the graph. The equation for the line is shown on the graph. Use the graph to answer the questions below.




Source: http://data.un.org/default.aspx

1. Describe the pattern in your own words.
2. Why is the gradient negative? What does that imply?
3. Find the point on the graph where the female enrolment rate is 100 or higher. What do you notice about the infant mortality rate?
4. Find the point on the graph where the female enrolment rate is lower than 50. What do you notice about the infant mortality rate?

You will explore more data in chapter 4.

# Tiny Home Design Investigation



*Many people are choosing to live in smaller houses to save on cost and simplify their lifestyle. Tiny Homes have been around for the past couple of decades and are designed to make clever use of small spaces. In this investigation you will examine the storage and living space in a tiny home design and compare it to your own home.*

## Brief

**Volume of storage space:**
Examine one design for a tiny home or caravan and calculate the total volume of combined storage space in the cupboards and storage areas using the elevations. Complete an elevation (front only) of one wall of your kitchen which has cabinets. Calculate the total storage space in those cabinets. Write a sentence or two comparing the total amount of storage in the tiny home with the cabinet space in one wall of your own kitchen.

## Skills Checklist

Skills you need to demonstrate for this assignment:

- Calculate area of composite shapes and sectors
- Calculate arc length
- Calculate volume of prisms and composite solids
- Calculate surface area of prisms and composite solids
- Draw and interpret scale diagrams and plans

**Area of living space:**
Examine one design for a tiny home or caravan and calculate the total floor area of the living and bedroom space (living room, bedroom and any veranda space). Draw a plan of your own bedroom and calculate its floor area. Use a scale of 1:50. Write a sentence or two comparing the total area of floor space in the living and bedroom of the tiny home to your own bedroom.

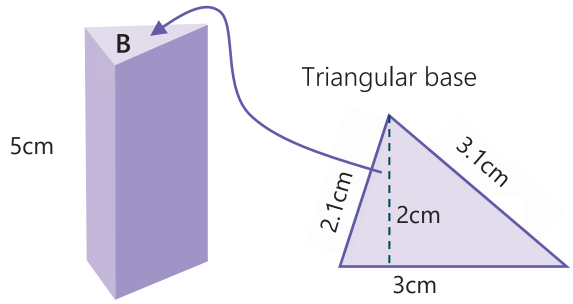
**Arc of door swing:**
Doors swing on arcs, making sectors. Using the dimensions given on the tiny home plan, calculate the arc length of the door and the area that it sweeps.

**Optional Complex Task: renovating a tiny home kitchen**
Imagine that you have found a tiny home available second-hand. It is cheap, but the kitchen cupboards and benches need to be re-laminated. Calculate the total area of laminate that you would need to recover one cupboard and one bench.

## Hand in

- Your calculations for volume of storage, area of living space, length of door arc swing and area of door swing. A paragraph comparing the tiny home to your house.
- A scale diagram of your bedroom with appropriate markings.
- The complex task is optional but will help you achieve a higher overall grade.

B. Triangular-based prism

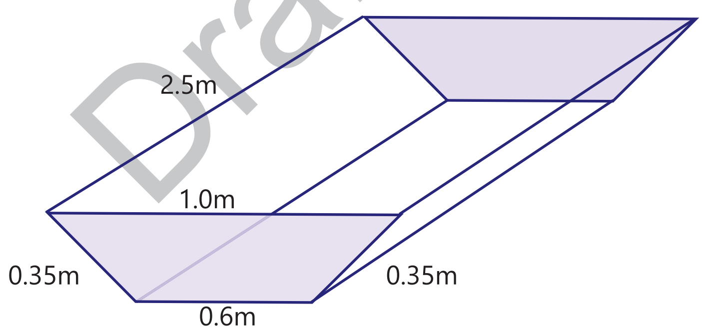



## Explain

How could you generalise the steps that you went through for the rectangular prism and triangular prism to apply to any prism? What could you do to figure out the surface area for any prism? Make sure that you consider any prism, not just ones with regular shapes for the base.

## Extend

Consider the drain pictured below. Its cross section is a symmetrical trapezium. The drain forms a prism, but the top is open to allow water in. It is 40cm deep. Draw a net of the drain and figure out the area of the surface that would touch water presuming that the drain was full.



# 28. Pyramid nets and surface area

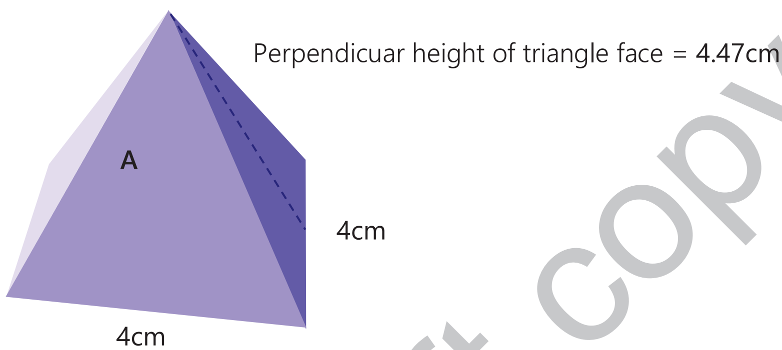
MULTIPLICATIVE THINKING   GENERALISING

*In the previous lesson, you worked out how to draw a net for any prism and calculate its surface area. In this lesson, you will apply the same thinking to nets and surface area of pyramids.*

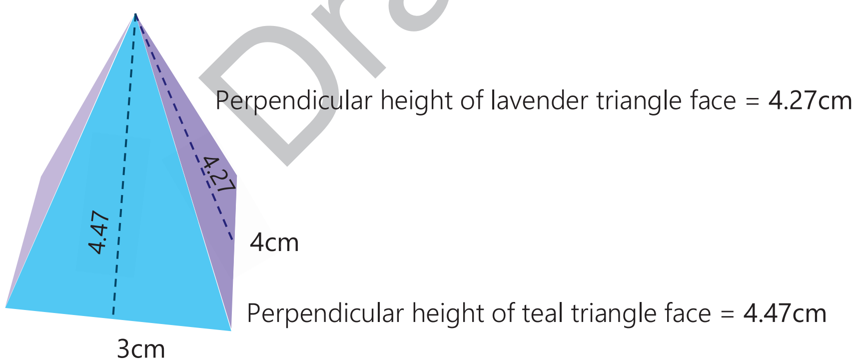
## Experiment

For each of the pyramids pictured below, (1) sketch its net and label the dimensions, and (2) use what you know about area of flat shapes to figure out the surface area.

A. Square-based pyramid. The square on the base is 4cm x 4cm.

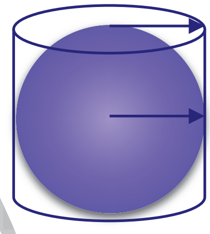



B. Rectangular-based pyramid. The rectangle on the base is 3cm x 4cm.



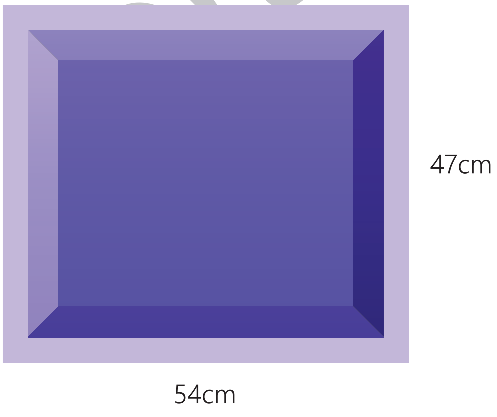

C. Tetrahedron: a triangular-based pyramid made of 4 equilateral triangles. The side length of each triangle is 4cm. The perpendicular height of each triangle is 3.46cm.

3. When we are considering the surface area of a sphere, we need to think about the cylinder that would surround it exactly. If we remove the top and base of that cylinder, the tube that is left has the same surface area as the sphere. If we folded that tube flat, it would make a rectangle. The length or base would be the same as the circumference of our sphere ($\pi$D). The height of the rectangle would be the same height as the sphere (its diameter). At that point we can use our Base by Height strategy to calculate the surface area pretty quickly. Use this thinking to calculate the surface area of the following spheres:
    a. Sphere with a diameter of 10cm
    b. Sphere with a radius of 4cm
    c. Sphere with a circumference of 20cm





4. The solid shape below is an open box, viewed from the top. The box is 54cm long, 47cm wide and 35cm high. Each wall is 2cm thick, so the rectangular faces inside the box are not the same dimensions as those on the outside. Imagine that we are re-laminating the entire box. Your job is to calculate the total surface area of the box, so that we know how much laminate will be needed. Remember to account for the base of the box and each surface. Show all your steps.



# 34. Calculate complex and composite volumes

MULTIPLICATIVE THINKING   PROPORTIONAL REASONING   GENERALISING

*Calculating volumes is very common in the building industry. In this lesson, you will calculate the volume of concrete needed for holding up poles for fences.*

## Investigate

An auger with a 150mm drill bit is used to dig holes for poles that are 150mm diameter. The poles to be inserted have a 100mm diameter. Each hole needs to be 600mm deep.

1. Calculate the volume of the hole and the volume of the pole inside that hole. Note: changing the measurements to centimetres first will allow you to easily change between cubic centimetres and millilitres/litres for the rest of this task.
2. Calculate the volume of concrete needed to surround the pole and fill the hole.
3. Convert the volume of concrete to litres and round off your calculations to one decimal place.
4. A 20kg bag of concrete powder mixes to make 10L of concrete. How much will you use of your bag of concrete?
5. How many bags of concrete would you need to fill 10 holes?
6. Sometimes, fences use a 75mm square pole instead of a round 100mm pole. How would this change the volume of concrete needed for one hole?

# Road Gradient Comparisons



*In September 2019, a new Toowoomba Bypass road was officially opened to traffic. The 41km long bypass cost the Australian and Queensland Governments $1.6 billion[1]. The bypass ensured that the maximum gradient of the road at any stretch was 6.5%, down from 10-14% for stretches of the previous Warrego highway[2]. The sign below shows a photograph taken of a particular section of the previous highway with a gradient of 10% over 4km.*


Source: https://www.expressway.online/oldsite/photogallery/roads/qld/numbered/nh-a2/index.htm

*In this investigation, you will compare the steepness of the previous Warrego highway with other steep roads and highways in your State.*

## Skills Checklist

Skills you need to demonstrate for this assignment:

- Interpret, estimate and calculate using scale drawings
- Interpret, sketch and construct plans and elevations
- Construct scale drawings
- Apply Pythagoras' theorem to calculate lengths in right angled triangles
- Apply tangent ratio to find unknown sides and angles in right angled triangles, including with angles of elevation and depression
- Apply cosine and sine ratios in right angled triangles

## Brief

You will create scaled elevation drawings of three steep roads: The previous Toowoomba range, the steepest road in your State and a highway in your state with a high gradient.

For each road, calculate the gradient, explain what that means in terms of rise for each horizontal metre of road, and convert your gradient to an angle in degrees. You also need to use a scaled map to calculate the horizontal length of each road, use the gradient to calculate the vertical climb of the road, then use these measurements to calculate the exact length of the road assuming a right-angled triangle is formed.



## Hand in

You will need to include the following information and cite your sources.

- A scaled elevation drawing of all three roads, labelled appropriately, including any measurements.
- Calculations for the gradient, angle and length of each road as described in the brief.
- An explanatory paragraph that lists the similarities and the differences between the roads, explains what the gradient means and provides any other useful information.

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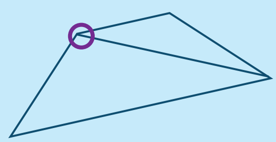
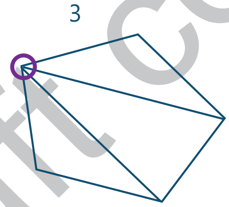
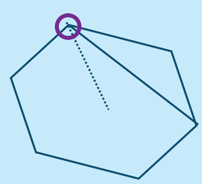
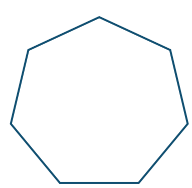
1 https://www.tmr.qld.gov.au/projects/toowoomba-bypass
2 https://www.roadsonline.com.au/tackling-the-toowoomba-range/

### Hexagons

1. What do you think the sum of the angles of a hexagon might be? How come?
2. Try out the procedure and check if your idea works. Were you correct? If not, what new pattern could you see that would fit your findings?

## Deduce and Infer

Complete the table below, deduce the patterns and explain your findings. Use the table to predict what you think might happen with the internal angles for other polygons.

| Polygon | Number of sides and angles | Number of triangles we could divide it into (NB. complete the diagrams) | Sum of internal angles | Relationship between internal angles and number of triangles |
|---|---|---|---|---|
| Triangle | 3 | 1 only | 180° | 1x180 = 180 |
| Quadrilateral | 4 | 2  | | |
| Pentagon | 5 | 3  | | |
| Hexagon | 6 |  | | |
| Heptagon | 7 |  | | |
| Octagon | | | | |

# 38. Pythagoras in quilting patterns

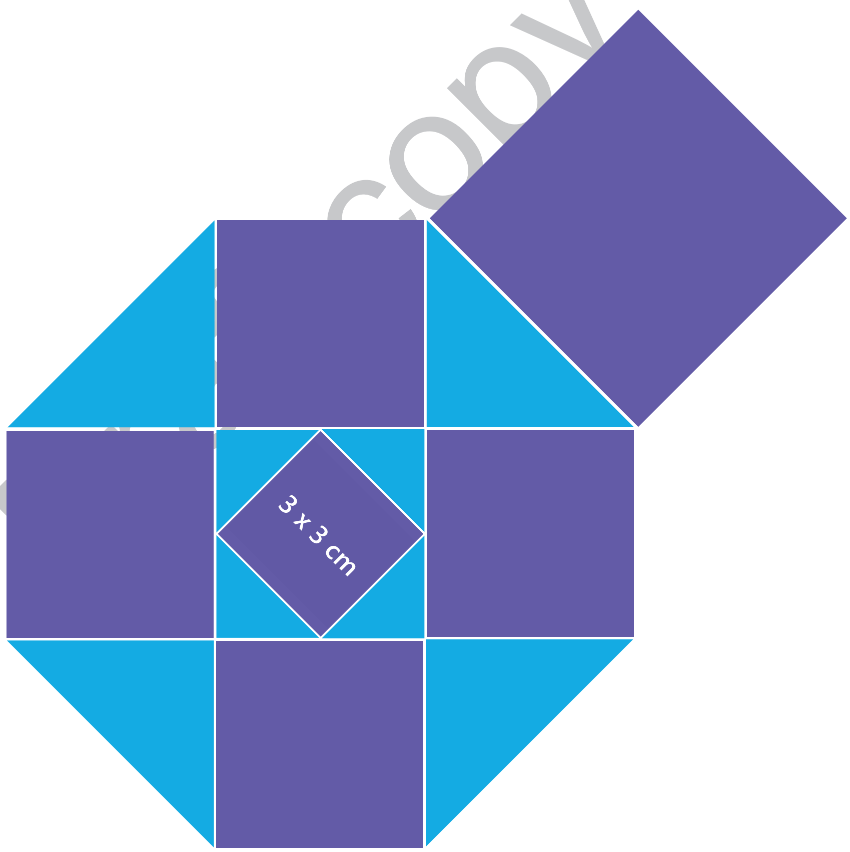
MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

*Quilters often use the Pythagorean theorem to resize quilting patterns. In this lesson, you will apply what you have learned to work out the dimensions of the fabric in the pattern shown.*

## Apply

Examine the quilting design below which is made of squares and right-angled triangles. Your job is to figure out the dimensions of the biggest square, assuming that the smallest square in the very centre is 3cm x 3cm. Each triangle can be assumed to be an isosceles triangle, with $45^{\circ}$ angles. Show all your working. Label each square or triangle as appropriate.

Please note: the pattern is not drawn to scale, so measuring the squares will not be helpful.

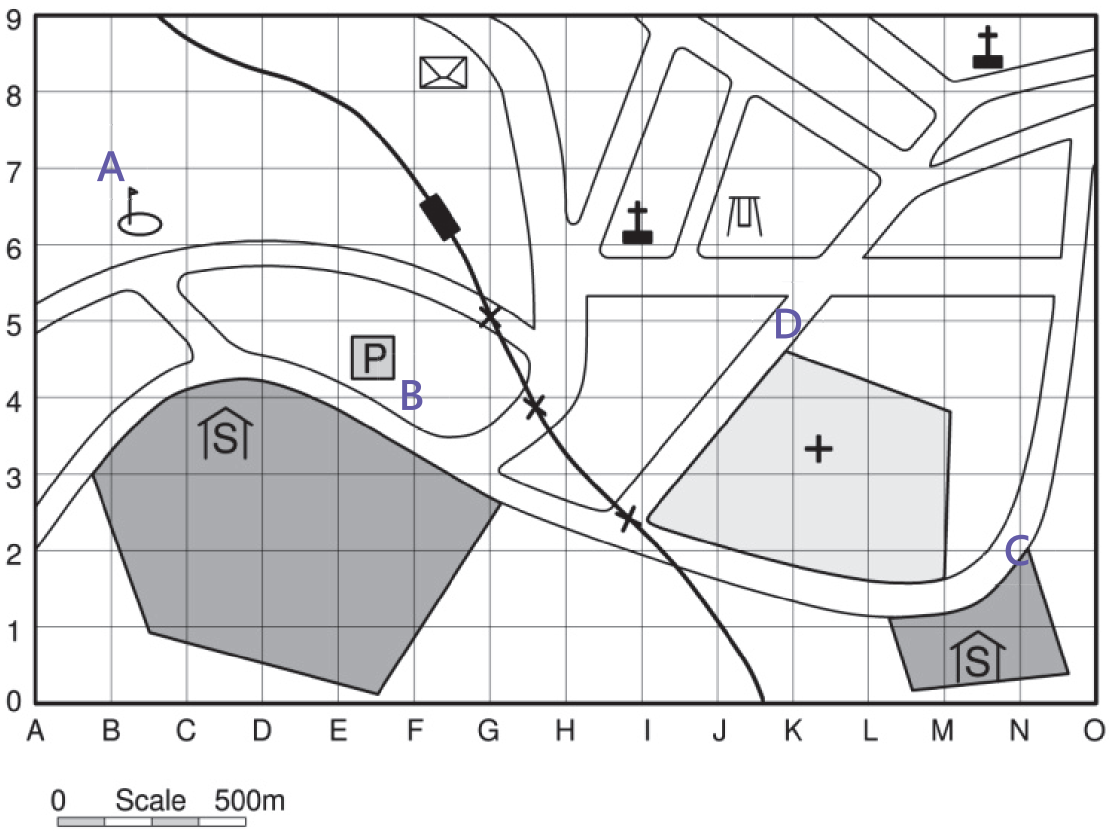


## 39. Pythagorean theorem and distance on maps

PARTITIONING    PROPORTIONAL REASONING

*In this lesson, you will calculate the distance between destinations "as the crow flies", but without measuring. You will need to apply your learning from the previous two lessons about the Pythagorean relationship between the side lengths of right-angled triangles to help.*

Consider a right angled triangle that would have vertices at A and B, with the right-angle at B4. Count the length of each side in squares, calculate the length in squares of the distance from A to B, then convert it to metres using the scale.

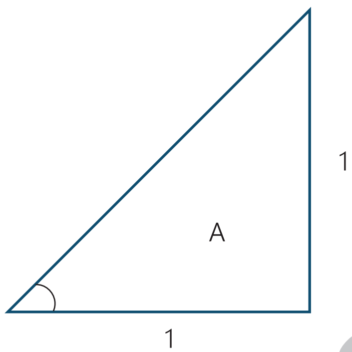


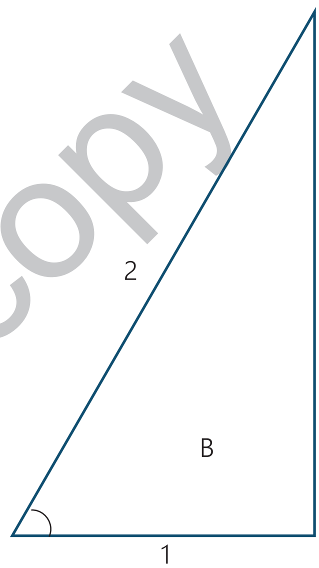

KEY

| | | |
|---|---|---|
| School | Railway crossing | Post office |
| Church | Railway station | Hospital |
| Police | Golf course | Playground |

## Extend

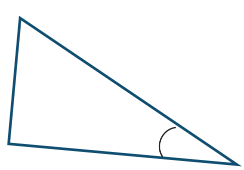
1. Calculate the distance between A and C.
2. The distance between points B and C can be calculated using the same method, however it will not have a whole number as an answer. To help you calculate the answer, you will need to use the square root button on your calculator. Ask your teacher to help you find the correct button as needed. Write your answer both as a square root (E.g., $\sqrt{3}$), and as an approximate decimal rounded to two places.
3. Use the same method to calculate the distance between D and C.
4. Use this same method to work out the lengths of the triangles shown below. These particular triangles are very important for our learning over the next few lessons. Write their missing side lengths both as a square root (E.g., $\sqrt{5}$) and as a rounded decimal number. Please note, the triangles have been scaled up to make them easier to work with.

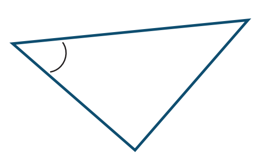





5. Measure each of the angles in the triangles above and write them in. Why do you think these triangles might be important?
6. Label the hypotenuse on each triangle. Draw in the right angle symbol for the appropriate angle.
7. Consider the angle marked for triangle A. Which side is adjacent to the angle indicated (touching it)? Label that side as "adjacent". Which side is opposite the angle indicated? Label that side as "opposite".
8. Repeat for triangle B.
9. Identify the right angle, hypotenuse, opposite and adjacent sides on the triangles below.





# 40. Gradient, ratio and calculating angles for right triangles

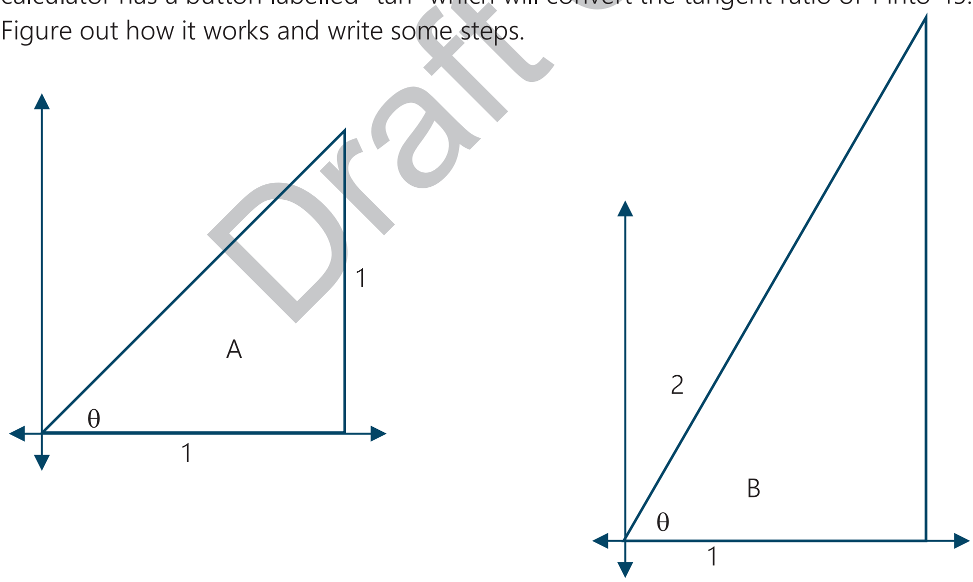
GENERALISING    PROPORTIONAL REASONING

*In the first chapter of this book, we calculated the slope or gradient of straight lines on axes using the equation: Gradient = rise/run. Gradient is a number that describes the angle made by a line. A bigger gradient made a steeper line and a smaller gradient made a flatter line. Gradient can be used to calculate an angle in degrees. Today we will link the two together.*

## Experiment

Below, we can see both triangles that we calculated the side lengths and measured the angles for in the previous lesson. Imagine that each of these triangles was drawn on an x-y axis, so that the hypotenuse of the triangle was almost like the shape of our previous line graphs.

1. Use the **Fractions are Division** strategy to **calculate the gradient** of the hypotenuse for A. Use the base length and height of the triangle as your run and rise. What is the gradient?
2. Label the hypotenuse on your triangle. Draw in the right angle symbol for the appropriate angle. Consider the angle marked with the $\theta$ symbol. Which side is adjacent to $\theta$ (touching it)? Label that side as “adjacent”. Which side is opposite $\theta$? Label that side as “opposite”.
3. Rewrite the formula for gradient, but substitute the words rise and run for “opposite” and “adjacent” sides of the triangle. This is known as the **tangent ratio**.
4. We know from measuring, in the previous lesson, that the angle indicated is 45°. We have calculated that the tangent ratio (opposite/adjacent) for the angle indicated is 1. Your calculator has a button labelled “tan” which will convert the tangent ratio of 1 into 45. Figure out how it works and write some steps.



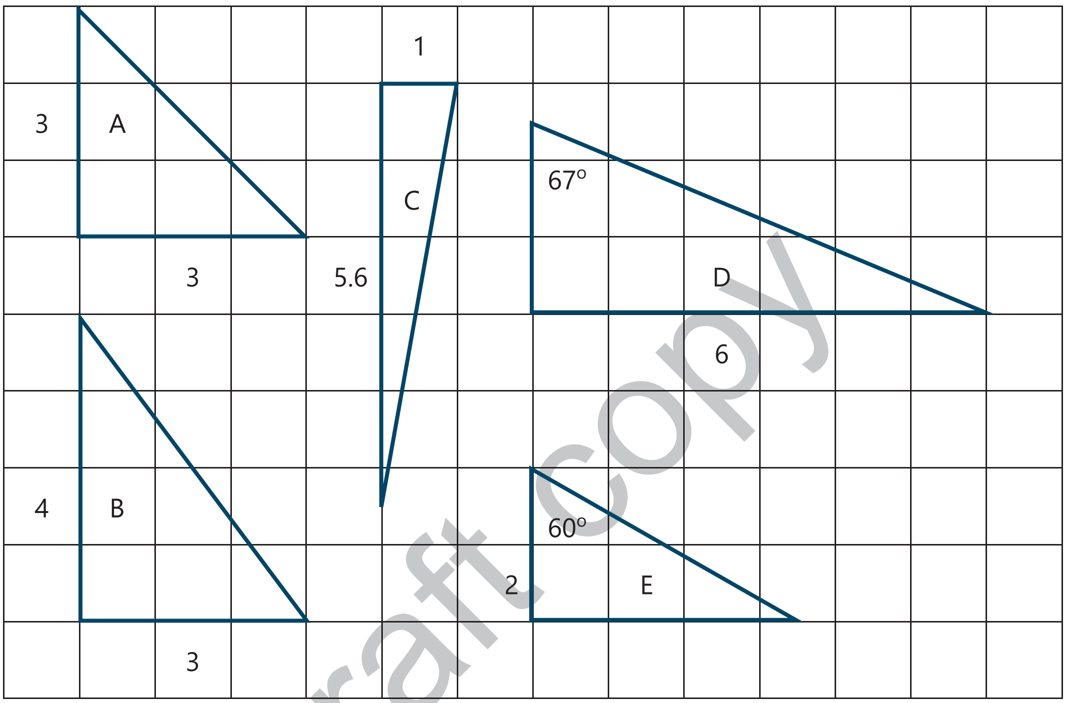

5. Identify the opposite, adjacent and hypotenuse sides on Triangle B in relation to $\theta$. Write the Tangent ratio for Triangle B as a fraction, still including the square root symbol. Show your working. Use the tan button to calculate the angle indicated.
6. We can also use tan to calculate the top angle in triangle B, but we have to relabel the opposite and adjacent sides first as they were in relation to the bottom angle. Relabel the sides, then use the tangent ratio to calculate the missing angle. Show your working.

## 41. Calculating side lengths and angles for right triangles

GENERALISING   MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

*We can use tangent in different ways: if we know the angle, we can figure out side lengths. If we know the side lengths, we can figure out angles. Find all unknown side lengths and angles below.*

### Practise

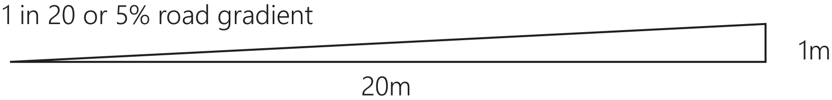





### Apply and Investigate

Road gradients describe the steepness of roads. They are calculated as rise/run, in the exact same way as gradients on line graphs. For example, a 5% gradient (or 0.05), means a rise of 5m for every 100m of horizontal distance. It is also described as a 1 in 20 gradient (1m rise for every 20m horizontal distance). A scale diagram is included below.

1 in 20 or 5% road gradient




1. Use tangent to calculate the angle of the road in degrees.
2. A steep road has a 1m rise for every 10m of horizontal distance (1 in 10 or 10% gradient). Calculate the gradient of the road and the angle in degrees.
3. The steepest public road in the US is Canton Avenue, Pittsburgh[3]. It has a gradient of 37%. Explain what this means and calculate the angle of the road. **Note:** *check your investigation.*

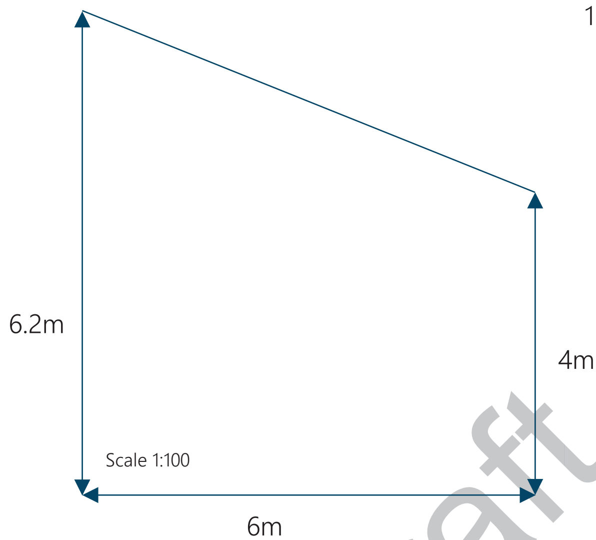
3 https://www.foxnews.com/travel/worlds-steepest-roads

# 42. Tangent, angle of depression and angle of elevation

GENERALISING MULTIPLICATIVE THINKING PROPORTIONAL REASONING

*The tangent ratio has many different applications in the real world. One of the most useful is measuring the angle of elevation or depression and using one known distance to calculate a length that is hard to measure. Another is using lengths that can be measured easily to calculate angles or lengths that cannot. In this lesson, we will apply the tangent ratio to a range of real world scenarios.*

## Apply the tangent ratio to solve problems

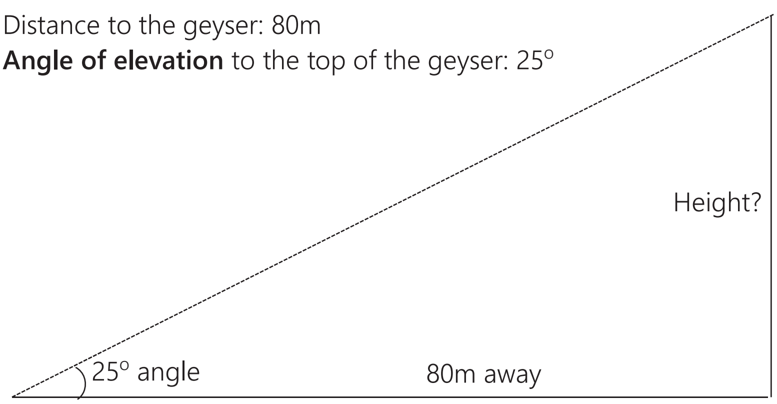



1. Output from solar panels on a roof is maximised when the panels are in full sun for most of the time. Our latitude describes the angle that the sun is in our sky half way between summer and winter. This means that our latitude also gives the angle that we want our solar panels to be on[1]. If our roof is on a similar angle, we can lay the panels flat on the roof. Examine the elevation diagram of a house at latitude 19°. Use the information provided to **calculate the angle** of the roof. Would you recommend that solar panels be placed flat on the roof? Show your working.



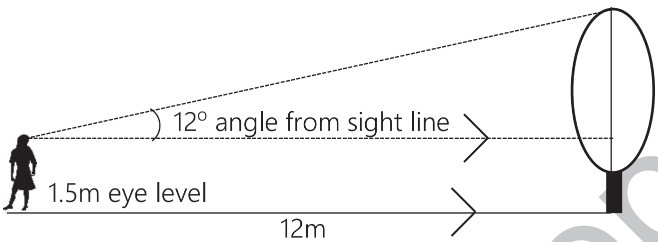
2. **Angle of elevation: look up at an angle**
   Sometimes measuring the height of something is very difficult, such as the geyser in this photograph that only reached its full height for a second. Instead, we can use an angle of elevation to figure out their heights. Determine the height of the geyser in this photograph using the information provided below and the scale diagram.

   Distance to the geyser: 80m
   **Angle of elevation** to the top of the geyser: 25°




3. **Angle of elevation:**
It is particularly difficult to measure the height of trees as branches tend to interfere with laser measurements. Instead, we can use an angle of elevation to figure out their heights, if we know how far away we are standing. Determine the height of the tree in this photograph using the information provided below and the scale diagram. You will need to take into account the height of the person for this problem as the angle of elevation was taken from eye level.

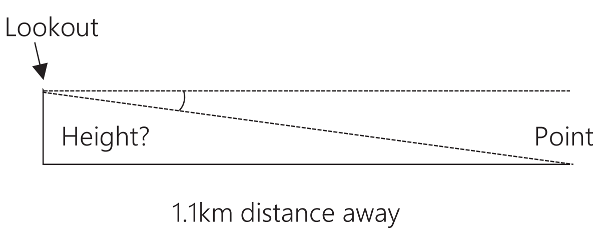
Distance along the ground between photographer and tree: 12m.
**Angle of elevation** from the camera to the top of the tree: 12$^{o}$.
Height of the camera above the ground: 1.5m.






4. **Angle of depression: look down at an angle**
At a lookout at the top of a hill I have this view. A nearby sign says that the Point I can see circled in the photograph is 1.1km away.
I measure the angle of depression as 8$^{o}$ below the horizon. How high am I above sea level?
Note: The diagram shows a horizontal line for the view to the horizon, with the angle of depression dropping down from the lookout to the Point.

**Angle of depression** to the point: 8$^{o}$.
Distance from the point: 1.1km.




**Note:** you should now be able to complete most of the calculations in your Investigation for this chapter.

# 90. Model interest-free periods

GENERALISING   MULTIPLICATIVE THINKING   PROPORTIONAL REASONING

*In this lesson, you will adjust your credit card model to include an interest-free period.*

## Investigate

Copy your credit card spreadsheet from the previous lesson into a new sheet so that you can easily make changes to the calculations. This time you will adjust the spreadsheet to model what happens when you have an interest-free period.
Assumptions:

- You spend $1000 at the start of January.
- The annual interest rate is 18%, with a 1-month interest-free period. Interest is monthly.
- You repay the amount owing at $100 per month.

When modelling an interest-free period, the easiest way is to calculate the interest charged after the initial repayment has gone out. This means that we need to move the repayment column and add another column to show the balance after repayment. Examine how the table below has been adjusted and make the relevant changes to your spreadsheet.

| | Start of month balance | Repayment | Balance after repayment | r per month (18% /12) | t (months) | Interest charged (P r t) | End balance (new balance + interest) |
|---|---|---|---|---|---|---|---|
| **January** | $1,000.00 | $100.00 | $900.00 | 1.5% | 1 | $13.50 | $913.50 |
| **February** | $913.50 | $100.00 | $813.50 | 1.5% | 1 | $12.20 | $825.70 |

1. Copy and drag the calculations down until the balance reaches close to 0. You will need to adjust the last repayment to pay off the loan so that the balance is 0, or very close to 0. How much did you pay altogether?

2. Complete the 18.0% with interest-free period column in the table below to compare the amount you pay to the rate calculated in the previous lesson. How much do you save?

| | 18.0% interest, No interest-free period | 18.0% interest, 1 month interest-free | 18.0% interest, 2 months interest-free |
|---|---|---|---|
| **Total** | $1090.27 | | |

3. It can be complicated to model what happens when we change the interest-free period. Try to work out how to change the interest to charge on the balance after 2 months of repayments. This should mean that you can just delete the interest from the first month. After that, the interest applies monthly. Complete the table above and compare the amounts you pay.

| | Start of month balance | Repayment | Balance after repayment | r per month (18% /12) | t (months) | Interest charged (P r t) | End balance (new balance + interest) |
|---|---|---|---|---|---|---|---|
| **January** | $1,000.00 | $100.00 | $900.00 | | 1 | $0.00 | $900.00 |
| **February** | $900.00 | $100.00 | $800.00 | 1.5% | 1 | $12.00 | $812.00 |

# Interleaved Question Sets

## Why this is important

In each of the following question sets, each question is different to the previous questions. That means you will have to think each question through, rather than simply implementing the same strategy. Sometimes this feels like hard work, but it is really good for our brains. It helps us to remember strategies for longer and be more flexible in our thinking. Each question set below gets a little harder. Use them in order, one set per week, so that you develop the necessary skills and strategies over time.

## What to do when you get stuck

The sets of questions are designed so that you won't know all the answers, or even have been taught all the content yet. This helps to stretch your thinking and develops your ability to work out something that you don't know how to do yet. When you get stuck, try the following approaches *before* asking your teacher for help. It is very important that you do the thinking for yourself first, as this will increase the chance of your learning being retained.

1. **Pair up** with another student. Read the question out loud, as that sometimes helps you to understand it better. Tell the other student what you think the question is asking rather than what you think you need to do. If you have time, split the questions between you, work out how to solve your set and then work out how to teach your partner the others. Teaching a concept to another person is a great way to develop a stronger understanding, and reciprocal peer tutoring can add 5 months of maths gain in a year to your results[47].
2. **Guess wrong first**: what is definitely too much? What is definitely too small? This will give you a range of reasonable possibilities.
3. Try a **strategy** that you have used for other questions. In this book we have taught you multiple strategies that all work across different chapters. Just try one and check.
4. **Look up the answer** in the answers section and then work backwards to figure out what process to use. This is an important part of deductive reasoning – making inferences from what we know.
5. Use the contents page from this book to **look up an activity** that teaches that concept or skill. Read it through to see if you can work out how to go about solving it.
6. If all else fails, **ask your teacher for a worked example**, then try again with a similar question. Let your teacher know where you are stuck so that they can teach that concept in a mini-lesson in the following week. Make sure that you do the thinking yourself as otherwise it won't stick.



[47] https://evidenceforlearning.org.au/the-toolkits/the-teaching-and-learning-toolkit/all-approaches/peer-tutoring/

## Set 1

1. Order these lengths from smallest to largest: 1.415m, 142cm, 145mm, 0.14km
2. Find the value of x in this equation: $2x - 1 = 9$
3. Write 0.65 as a fraction and a percentage.
4. Find the area of a rectangle that is 24cm long and 8cm wide.
5. Find the perimeter of the same rectangle.
6. I pay $83.56 each week for my car loan. Approximately how much will I pay over the period of a year?
7. Bearings are angles that are described as a clockwise rotation from North. What would the bearing be for South-East?
8. What would the bearing be for North-West?
9. Sectors are formed by slicing circles into parts through the centre. A sector had an angle at the centre of $90^{o}$. What proportion of the circle's area would it take up?

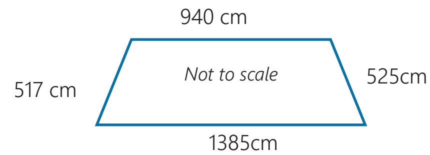
## Set 2

1. A statistician wanted to sample 30% of a target population. If the target population was 12 000 people, how many should be in the sample?
2. A walking map used a ratio of 1:10000. If the campground is 12cm from the creek on the map, how far is it in real life?
3. What fraction of $2 is 45 cents?
4. The population[48] for Queensland in December 2018, was 5 052 800, for New South Wales 8 046 100 and for Victoria was 6 526 400. How many NSW residents are there for every 1 QLD resident?
5. If the Australian population is 25.18 million (December 2018). According to the ABS, Australia's population grows by about 1.6% per year. If the population was 25.18 million in 2018, what will it be this year? You will need to either use multiple steps or work out how to use compounding growth with the rate of 1.016 from Lesson 84.
6. Draw a triangular prism.
7. Draw a net for the triangular prism.
8. How many faces, edges and vertices does a triangular prism have?
9. How could you use the net of the triangular prism to calculate its surface area?



[48] https://www.abs.gov.au/AUSSTATS/abs@.nsf/allprimarymainfeatures/1988DE98D5424933CA258479001A75A5?opendocument

## Set 3

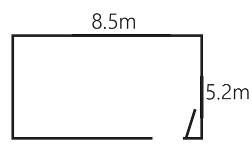



1. What is the perimeter of this shape?
2. Sketch a set of axes with the points (0,2) and (2,4).
3. I have a 60% chance of winning a sporting match. Describe what this means.
4. Find the value of x in this equation: $x^2 - 1 = 8$
5. The vehicle registration cost[49] for my 4-cylinder car in 2021 was $744.45 for 12 months. I have had my car for 4 years and 3 months. Approximately how much have I paid for registration so far?
6. If my car had 5 or 6 cylinders, it would have cost $939.60 for 12 months registration. How much more would this car cost to register over the same time period?
7. A triangle has an area of $12m^2$. Give 3 possible sets of dimensions for base and height.
8. Bearings are angles that are described as a clockwise rotation from North. What direction would have a bearing of $225^o$?
9. Sectors are formed by slicing circles into parts through the centre. A sector had an angle at the centre of $60^o$. What proportion of the circle's area would it take up?



## Set 4

1. Sketch a set of axes with the points (0,3) and (3,6). Draw a line to join the points. Write the equation for the line.
2. A builder is fitting wood panelling on each wall of a rectangular room. The door is 1.2m wide and doesn't require panelling. Panelling can only be bought in whole metres. How many metres of panelling should the builder buy?

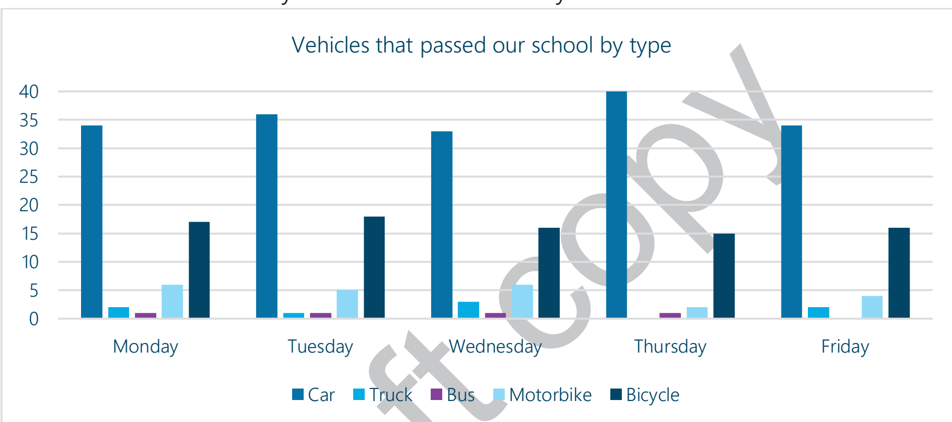



3. The door swings in an arc. If the door swings open to an angle of $90^o$, how could you work out the length of the arc? (see Lesson 18 for help)
4. I spent $35 out of my $90 budget on a shirt. What fraction of my budget did I spend? What percentage of my budget did I spend?
5. If it is 8pm in QLD, what time will it be in WA? (see Lesson 50 for help)
6. Sketch a triangular pyramid, with regular triangles for each face.
7. Draw the net for the pyramid. How could you use the net to calculate the surface area if you knew the base and height for each triangle?
8. A tower was made of blocks. Each layer was identical. There were 10 layers altogether. The bottom layer had 12 blocks. How many blocks were in the tower?
9. A house plan used a ratio of 1:1000. If the bathroom is really 3.1m long and 2m wide, how big is it on the plan?

[49] https://www.qld.gov.au/transport/registration/fees/cost#common

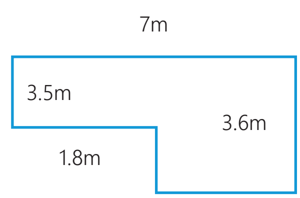
## Set 5

1. Write these capacities in order from smallest to largest: 12.605kL, 12 065L, 12 560 741mL, 12.506kL
2. Convert each of the capacities from the previous question into cubic cm or cubic m.
3. If a square had a perimeter of 70 centimetres, how long was each side?
4. Find the value of x in this equation: $2x^2 – 2 = 30$
5. Sketch a line graph that matches the equation $y = 2x - 2$
6. What 2D shapes have 2 sets of parallel lines and 4 sides, but are not squares or rectangles?
7. What is the bearing for NNE?
8. The following graph shows vehicles driving past our school each day for one week. Calculate the ratio of bicycles to cars on Thursday.






## Set 6




1. This is a drawing of the coop I am building for my chickens. Calculate its area.
2. Calculate its perimeter.
3. Simple interest was covered in Year 11, and is also reviewed in Lesson 80. What would the final value be of an initial investment of $200, invested for 3 years at an 8% simple interest rate?
4. I fly from Perth at 8:55am and land in Brisbane at 17:35. How long was my flight?
5. Sketch the equation $y = 1/2x + 3$
6. A road rose 5m in height for every 100m in length. Sketch the road on the correct angle. Use a scale diagram to help.
7. The exchange[50] rate European euros was 0.6085 Euros (symbol is €) for every Australian dollar. If I exchange $1200AUD for Euros, how much will I receive?
8. If I exchange €500 for Australian dollars, how much will I receive?
9. What are the coordinates for the Adelaide GPO? Which is the longitude and which is the latitude? (See Lesson 48 for help)

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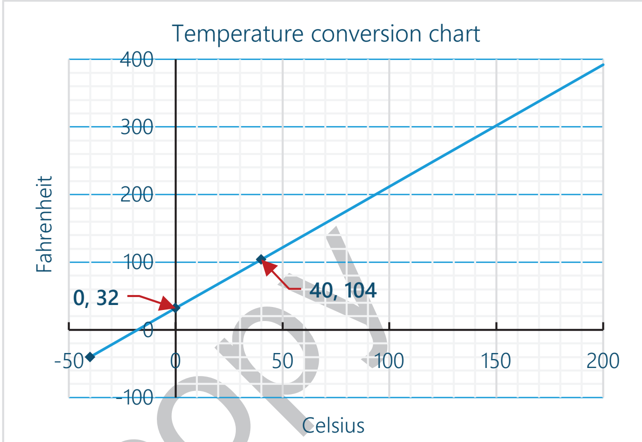
[50] https://www.rba.gov.au/statistics/frequency/exchange-rates.html

## Set 7

1. My swimming pool is a rectangular prism. It has a *perimeter* of 18 metres. It is 3 metres wide. It is 1.5m deep. What is its volume?
2. I need to score at least 75% for a test. The test has 45 questions. How many do I need to get correct?
3. My car uses 1L fuel for every 14km. If I used up 5L of fuel, how far did I drive?
4. If I drive 200km, and petrol costs $1.80 per litre, how much do I spend?
5. The temperature conversion chart graph to the right is a straight-line graph. Write an equation for the line, that you can use for converting Celsius (x) into Fahrenheit (y).




6. Using your equation, what is the temperature in F of 50°C?
7. Using the graph, what is the temperature in C of 50°F?
8. A circle has a radius of 1.2m. What is its circumference? (See Lesson 17)
9. Are you more likely to randomly draw a face card or a numbered card from a deck of cards? How can you use the number of cards to work it out?

## Set 8

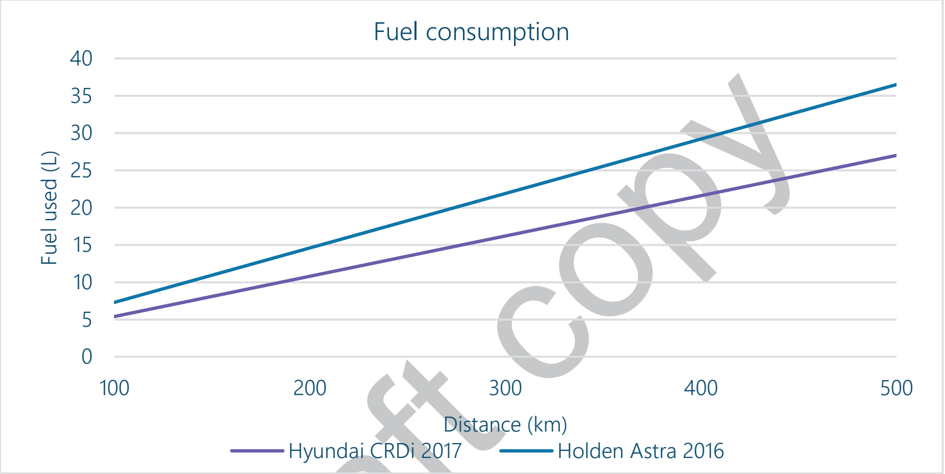
1. What square factors do the numbers 18 and 36 have in common?
2. A megalitre (ML) is 1000 kilolitres, or 1 million litres. The Ross River Dam near Townsville in QLD has a capacity of 233 187ML[51]. In May 2021, the dam was 75% full. How much water was the dam holding?
3. During the drought in 2016, dam levels dropped to 14% before pumping from the Burdekin dam began. According to the Townsville City, the target for daily water use is 100ML or less. Using this target, how long would the water in the dam have lasted?
4. Sketch a graph of the equation y = -2x + 1.
5. A sector has an angle of 135°. If the length of the arc is 20cm, what is the circumference of the whole circle?
6. What is the area of the whole circle?
7. A spinner had a 50% chance of landing on red, a 10% chance of landing on green and a 40% chance of landing on blue. Draw the spinner.
8. Is latitude or longitude more likely to affect your climate? Explain your answer.
9. An online meeting is set for January 3rd, at 9:00 Sydney time. What time would the meeting be for someone working in Brisbane, Adelaide and Perth?



51 https://www.townsville.qld.gov.au/water-waste-and-environment/water-supply-and-dams/dam-levels

## Set 9

1. Draw a net for a cylinder to scale. The cylinder has a base has a radius of 2cm and a height of 5cm.
2. The Perth stadium is at 42% capacity by 1 hour before the game. The Perth stadium holds 60 000 people. How many seats are unfilled?
3. Find the value of x in this equation: $2x^2/3 = 6$
4. I pay 19% tax[52] for every dollar I earn over $18 200. How much tax will I pay if I earned $38 000 this year?
5. I travel 210km in 3 hours. If I needed to drive 380km, how long would it take?
6. The following graph shows fuel consumption. Write an equation for the fuel consumption for the Hyundai.




7. Write an equation for the fuel consumption for the Holden.
8. Write a rate for the fuel consumption for of the Holden compared to the Hyundai (1 to another number - where 1 is the Hyundai with the other number is the Holden).
9. What are the possible totals for rolling two dice? (E.g., could be a 1 and 1, so total of 2)



## Set 10

1. A sector has an angle of 120°. If the length of the arc is 20cm, what is the circumference of the whole circle?
2. What is its radius of the circle from the previous question?
3. Write $\frac{19}{20}$ as a decimal.
4. A right-angled triangle has an area of $12m^2$. What could its base and height be?
5. A trapezium has the same area as the triangle. Draw a trapezium and give the relevant dimensions to make it work.
6. I invest $500 for 5 years at a simple interest rate of 2.5% per annum. How much do I have at the end of 5 years?
7. Sketch a 3D solid with 5 faces.
8. Draw a net for a different type of 3D solid with 5 faces.

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[52] https://www.ato.gov.au/rates/individual-income-tax-rates/#Residents

## Set 11

1. Write an equation to calculate the total cost for this problem: School students went on an excursion. The bus cost $4.75 for each child. The food cost $120 altogether.
2. Create a table of values from the situation above showing costs for 5, 10, 20 and 50 children.
3. Identify the gradient of the line from your previous equation.
4. At the start of the year, I invested $200. At the end of the year, I had $210. What was my simple interest rate?
5. Find 30% off $60
6. The hour hand on the clock shows that it is 8pm. Give the bearings for the hour hand.
7. $10 = 2x^2 - 40$ What is the value of x?
8. Perth Temperature data, 1994-2020 is provided in the table below. What was the mean maximum temperature across the whole year?

| | Jan | Feb | Mar | April | May | June | July | Aug | Sept | Oct | Nov | Dec |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean maximum temperature | 31.2 | 31.6 | 29.6 | 25.9 | 22.3 | 19.5 | 18.4 | 19.1 | 20.4 | 23.4 | 26.8 | 29.3 |
| Mean minimum temperature | 18.1 | 18.4 | 16.8 | 13.8 | 10.4 | 8.6 | 7.9 | 8.3 | 9.5 | 11.6 | 14.4 | 16.4 |

Source: Australian Bureau of Meteorology http://www.bom.gov.au/climate/averages/tables/cw_009225.shtml

## Set 12

1. Jordan borrows $2000. The payday lender charges 20% of the total in "set up fees". How much will Jordan have to pay back before any other charges?
2. The payday lender is allowed to charge an annual "account administration fee" of 4%, charged monthly. What percentage will Jordan be charged on his balance each month?
3. In the first month, Jordan pays back $200 of the amount he owes. The payday lender then adds the "account administration fee". How much does Jordan owe at the end of the first month?
4. How much does Jordan owe at the end of the second month?
5. What is the chance of rolling a 5 or greater on a 6 sided die?
6. Sketch an elevation and a plan of a hexagonal-based pyramid.
7. A tennis ball has a circumference of 21cm. What is its radius?
8. In 2014, 5.8 million Australians participated in voluntary work[53]. 42% of the volunteers were aged 15-17 years. How many volunteers were not 15-17 years old?
9. Sketch a net for a rectangular prism with dimensions of 3x4x5cm and calculate its surface area.



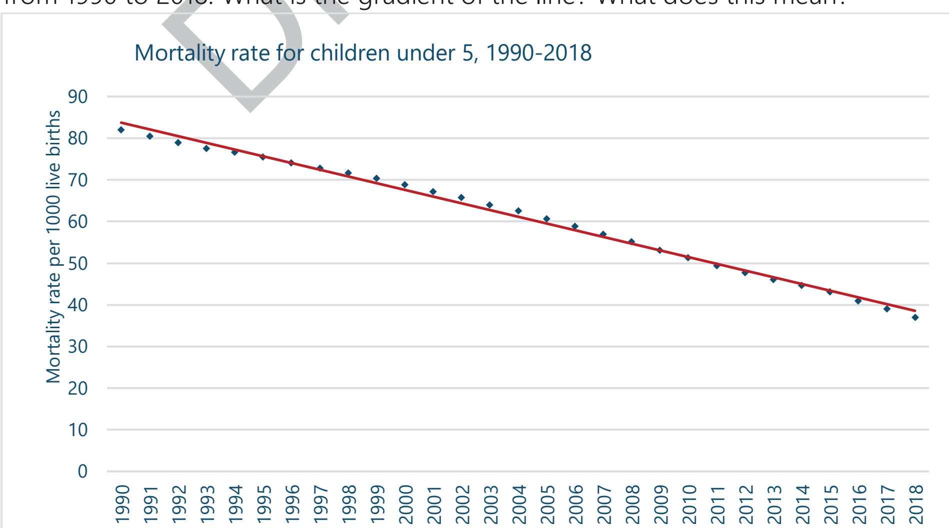
[53]https://www.abs.gov.au/ausstats/abs@.nsf/Latestproducts/4159.0Media%20Release102014?opendocument&tabname=Summary&prodno=4159.0&issue=2014&num=&view=

## Set 13

1. List all factors of 72.
2. After 1 July, a pay increase of 1.75% will apply to my wages. I usually earn $170 per week at my part-time job. Approximately how much more will I earn after 1 July?
3. I travel 100km at 80km/h. How long will it take me to arrive?
4. A cone has base with a radius of 3cm. It stands at 10cm high. What is its volume?
5. Write $\frac{3}{8}$ as a decimal and a percentage.
6. $x^3 - 4 = 23$ What is x?
7. 3 chocolate biscuits are baked for every 2 choc-chip biscuits. The baker has made 530 biscuits. How many choc-chip biscuits are baked?
8. Am I more likely to throw two heads/tails in a row, or 2 heads/tails separated?
9. A ball has a diameter of 16cm. Calculate its volume.

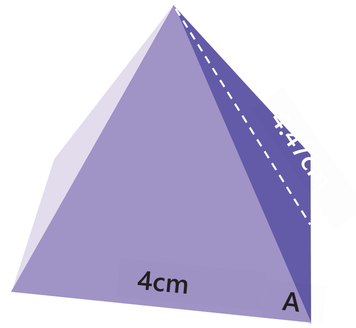
## Set 14

1. Consider the numbers from 15 to 35. Which are prime? Which are composite?
2. A shirt was marked down from $30 to $24, then reduced by another 20%.
   What was the final cost, and what percentage was the overall discount compared to the original price?
3. I tossed a coin 3 times. List all the possible outcomes (E.g., HHH).
4. What fraction of the outcomes have at least 2 heads?
5. Draw a rectangle and mark on all its properties (E.g., parallel lines).
6. A trapezium has an area of $20m^2$. What could its dimensions be (length of each of the parallel sides and height)?
7. A cylindrical can of drink has a volume of 445mL. What could its dimensions be?
8. The following graph shows decreasing mortality rates globally for children under 5 from 1990 to 2018. What is the gradient of the line? What does this mean?




Source: https://www.gatesfoundation.org/goalkeepers/report/2019-report/#ExploreTheData

## Set 15



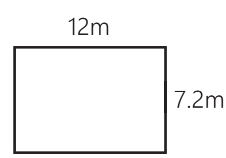

1. The square-based pyramid shown has a length of 4cm. The dotted line forms a right angle with the base of the triangular face. Calculate the area each triangular face.
2. Calculate the surface area of the pyramid.
3. Use Pythagorean theorem to calculate the length of the edge of the triangular faces (the base is 4cm).
4. Consider what it would look like if a line was drawn directly through the pyramid, from the apex to the base at 90°. This would be the height of the pyramid. It would form a right-angled triangle with the dotted line and half the length of the base. How high is the pyramid?
5. Use the height from question 4 to calculate the volume of the pyramid.
6. Consider a pack of cards. What is the chance of drawing a card with a value of less than 7? Write it as a fraction.
7. What is the chance of drawing a card with a value of more than 7? Write it as a fraction.
8. What is the chance of drawing a card with a value of exactly 7? Write it as a fraction.
9. If you add the fractions together from questions 6-8, what is the total?

## Set 16

1. In the pyramid pictured above, what would be the value of angle A?
2. 402 ÷ 6
3. Write $4\frac{3}{20}$ as a decimal.
4. How much will I pay for a $75 item that is marked 20% off?
5. The following pre-test and post-test results were obtained by 16 students. Which set of results should go on the x-axis? Explain your reasons.

| Pre-test | 114 | 117 | 118 | 120 | 121 | 122 | 122 | 123 | 123 | 125 | 126 | 126 | 126 | 128 | 128 | 131 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Post-test | 128 | 126 | 125 | 129 | 133 | 116 | 129 | 128 | 125 | 121 | 133 | 121 | 125 | 126 | 133 | 133 |

6. The gradient of a new road was 10%. Explain what this means and include a diagram.
7. I walk 4km East, then 3km North. How far away from my starting point am I now?
8. My garden bed is pictured to the right. I want to plant fruit trees in my garden in a grid-like pattern to maximise their production. Each tree will grow to have a radius of 1.2m. How many fruit trees could I plant in the garden? Draw a diagram to help explain.




9. What area will the trees take up altogether?