WHAT STUDENTS NEED TO KNOW ABOUT NUMBER

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Introduction

The following sections contain information about what your students need to understand about number, what happens when they do not understand, how to recognise this, and what to do about it. This section is largely derived from our observations in classrooms, which subsequently aligned with the findings of Big Ideas in Number, and includes similarities to some information contained in First Steps in Mathematics. Readers interested in learning more about critical concepts in mathematics would be well served to make reference to these useful publications.

Important understandings developed in the primary years of schooling include:

1. Identifying and writing numbers using a variety of representations
2. Counting and ordering numbers
3. Comparing numbers
4. Ordinal numbers
5. Renaming or partitioning numbers
6. Fractions
7. Performing simple operations

Identifying numbers

Common difficulties or misconceptions:

- Young students may be able to identify numbers of objects using counting or subitising (the ability to 'see' how many there are in a small group without counting) techniques but may need help to link this to the numerals we use to represent them.
- They may not understand that when a group of objects are rearranged or counted in a different order, the number remains the same.
- They may not be able to recognise numerals without linking them to a group of objects.

These understandings develop over time and are an important focus of number work in the early years.

Counting and ordering numbers

Concepts students need to understand:

- The purpose of counting is to find out how many there are
- Each object must be counted only once
- Numbers are said in a particular order
- Objects can be counted in any order and no matter which order you count them in, the number of objects will remain the same
- The way in which objects are arranged does not affect how many there are
- The last number said tells how many objects there are
- There is no need to count when the number can be easily recognised (subitising small numbers of objects)
- Sometimes it is easier to use skip counting to count larger collections (e.g. counting in twos)

- Skip counting a group of objects will tell us how many there are in a group.

Strategies to help develop understanding:

- Saying number names in familiar rhymes and songs
- Reciting the number sequence, first to 10 and then to 20
- Reciting numbers starting from a given number (may be a number other than one)
- Saying what number comes after/before a given number
- Starting the count at a given number and stopping at another number (forwards and backwards)
- Counting on or counting back a set number from a specified number (e.g. count on three more after 4)
- Including zero in counting sequence, particularly when counting backwards
- Counting:
    - Objects that are alike, different or placed in various positions
    - Objects in different orders (e.g. start with the shell)
    - A specified number from a larger group of objects
    - The marks on a number line or squares on a number track
    - Things other than objects (e.g. steps taken, claps, jumps)
- Recognising a small number of objects, up to six, without counting them (subitising). Items may be placed in random patterns or easily recognised patterns (e.g. dice or domino patterns)
- Guessing (estimating) the number of items in a collection and then verifying by counting. Discussing the accuracy of their guess using language such as too many, not enough, close, nearly...
- Recognising when counting is needed (e.g. when getting enough pencils from the container for everyone in their group)
- Arranging collections of objects in order of size
- Skip counting collections of objects.

Comparing numbers

Young students usually understand number to be a way of describing a set. This set may be objects, movements, sounds etc. As their understanding of number develops, they may be able to think of a number as an entity in itself and can compare this number with others without having to link them to a set. They become able to make statements about the relative magnitude of numbers e.g. 7 is bigger than 2.

Strategies to help develop understanding:

- Locating and positioning numbers on number lines and jumping forwards and backwards.
- Calibrated number lines (which detail a section of number line) encourage students to think about the relative magnitude of numbers.
- When the calibrations vary, students have to estimate where particular numbers would be located. (e.g. Locate 3 on a number line that has only 0, 5 and 10 marked on it)

Ordinal numbers

Students need to be able to see the relationships between cardinal and ordinal numbers. Cardinal numbers are referring to quantity, or the number of objects in a set. Ordinal numbers describe the position that objects take in relation to the others. Ordinal numbers only become relevant when the sequence of the objects is important. When counting objects in a particular order, students need to understand that the object they assign the name ‘six’ to is the sixth one that they have counted and that there were five things before it.

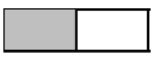
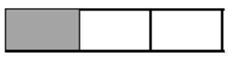
Renaming or partitioning numbers

Fractions

Students will need extensive practise partitioning a range of either discrete or continuous wholes into equal sized parts. Discrete wholes (groups of individual things that cannot be partitioned into smaller parts, e.g. groups of people) require dealing out or distributing the items to make equal parts. Continuous wholes (one thing that can be partitioned into equal sized parts) include:
- Area models in which partitions can be drawn, cut or folded.
- Linear models (e.g. number lines, lengths of tape) in which partitions can be cut or drawn.
- Volume models (e.g. containers, lumps of dough) into/out of which a partition can be poured or cut and measured.

Concepts students need to understand:
- When we partition something into equal parts, it has to be completely used up.
- The size of the whole stays the same no matter how it is partitioned.
- A half refers to the size of the portion rather than what it looks like. The two halves may look different even though they are equal in size.
- The more partitions we make, the smaller these partitions will be.
- A half of something small will be smaller than a half of something big.
- Later, students develop an understanding of relationships between fractions e.g. 1 quarter of something is more than 1 fifth of it.
- When a given fraction has been partitioned into smaller partitions, equivalent fractions can be seen.
- 3 quarters of a collection, object or quantity can be thought of as 3 out of every 4 parts.

Common difficulties or misconceptions:
- Some students may begin by associating halves with sharing of any sort and the idea of fairness. Everyone must have the same share regardless of the number of people sharing.
- Some students will label any partition of two as a half, regardless of whether they are equal.
- When something is partitioned into halves, students may think that both halves have to look the same.
- When students are sharing/partitioning a whole, they may neglect to use up all of it in the partitions and may discard the remaining portion.
- Some students may think that the ‘whole’ must be a single thing.
- Students may confuse fractional representations with those indicating ratio:

i.e.  confused with 1:2 

Strategies to help develop understanding:

Students should:

- Make their own partitions initially using concrete objects and other continuous models.
- Link the action of sharing or partitioning into equal portions to fractional language, using unit fractions (one half, one quarter) before using non-unit fractions (three quarters, four fifths) or linking the language to written representations.
- Be able to find one half of something by separating it into 2 equal parts, find one half of a volume by half filling a container or partition a group of objects into two equal parts and naming each part as a half.
- Students need to be able to:
- Identify what the whole is
- Make sure that the partitions are equal
- Use a fraction name that is related to the number of equal parts (e.g. quarters – 4 equal parts)

As their understanding develops, student should:

- Be able to count in fractional amounts e.g. one half, one, one and one half...
- Be able to relate mixed numbers to actual quantities e.g. 3 halves is the same as 1½
- Be able to interpret non-unit fractional quantities both multiplicatively (3 quarters is 3x¼) and additively (3 quarters is ¼+¼+¼)

Note:

Symbolic representations should not be introduced until students are able to use and understand fractional words.

Students may forget what the whole was, once they have partitioned it or cut it up into equal parts. It is important that their attention is drawn back to the whole that they started with by displaying a copy of the whole for them to look back on and to use it to compare with the partition/s they have made.

Performing simple operations

Concepts students need to understand:

- Numbers have their own meaning (e.g. understand that four has a value whether it is attached to a group of objects or not).
- Numbers have a constant value in operations and do not require a link to real world objects (e.g. 3 + 4 will always give you 7 and the numbers can be rearranged or counted in different ways but still give you 7).
- Facts can be related (e.g. 3 + 4 = 7 so 3 + 3 would equal 6).
- Partitioning numbers helps with difficult calculations (e.g. 6 + 5 is the same as 6 + 4 +1 or 10 + 1).

Students need to be able to:

- Picture small numbers in their heads (visualise) and groupings that make up these numbers e.g. 5 can be 2 and 3 or 4 and 1
- Use hands-on materials to represent a problem and join (addition) or separate (subtraction) them to solve the problem.
- Act out a problem or draw pictures to represent a problem in order to solve it.
- Visualise numbers and use mental counting strategies to solve simple addition and subtraction problems.
- Select addition or subtraction, as appropriate, to solve problems involving small numbers.
- Represent problem situations dramatically, physically, diagrammatically, verbally or symbolically.
- Use a variety of everyday language to express addition and subtraction situations.

- Understand the effect that addition and subtraction have on whole numbers e.g. subtraction makes whole numbers smaller and addition makes them bigger.
- Use appropriate language to describe situations multiplicatively (e.g. there are two groups of/lots of three cars). This is preferable to thinking of multiplication situations as repeated addition (e.g. 3+3) as it may become difficult to change students' thinking when they are introduced to the algorithm.