Issue 12
Aug 2014

The Insightful Classroom

The Intervention Issue

- The 8-lesson intervention project for Education QLD
- Extract from "Fixing misconceptions in fractions" book
- A word to HOCs: An extension box

Regular Insights, Tips and Pointers for Australian Maths Teachers

The Intervention Issue

What to do when nothing is working

>>> *The eight lesson intervention project*

Education Queensland's Pilot Project yields dramatic results

In late 2013, Education Queensland ran a pilot project aimed at improving student results rapidly in both Place Value and Fractions using problem-based teaching. Teachers from three schools were given brief professional development and written resources to teach just eight critical lessons to achieve dramatic improvements in student understanding. Here's what happened…

Principals gave their teachers permission to ***replace eight lessons from Curriculum into the Classroom (C2C) with 8 lessons from Back-to-Front Maths (B2F)***, implemented at any time during a ten-week period between 3rd and 4th term 2013. Gains were measured using matched pairs of NAPLAN questions, and compared with a control group of 5 classes which used C2C for the entire period. EQ have given permission to publish the results for the 14 classes, but the region, the schools and the teachers from the project be de-identified.

Results:

In the figures below, Group A students (121 students) used C2C for the entire 10 week project. Group B (96 students) had 8 of the 50 C2C lessons replaced by B2F lessons over the course of 10 weeks. Group C (94 students) had as many C2C lessons as the teachers chose replace d by B2F lessons (typically 2-3 lessons per week over the 10 week period). Details on what was tested and how this growth was achieved follow.

Growth in average student score between the pre and post test	Lower Primary	Upper Primary
Group A (121 students – all lower primary)	7%	No students in this category
Group B (96 students)	13%	18%
Group C (94 students)	26%	33%

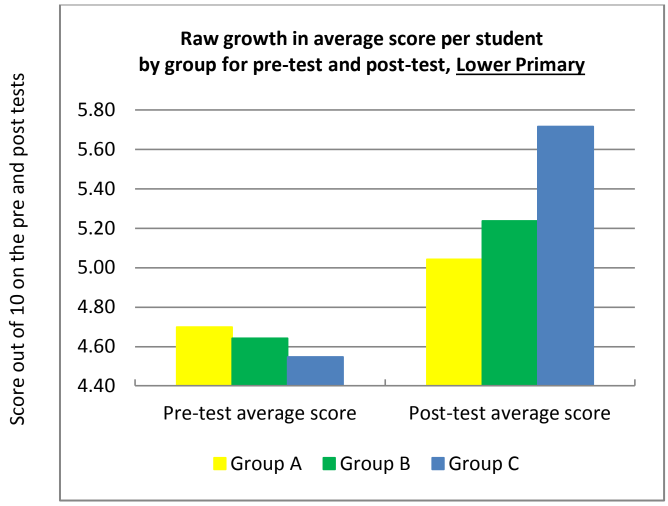


Figure 1 (left) shows the average score for the pre- and post-test per student by group for the **lower primary** classes.

Each group started off with fairly similar results. While some growth is evident in the C2C group (7% - effect size 0.3), much higher growth is evident in the students who were given eight B2F lessons (13% - effect size 0.6), with the growth for the B2F group being almost four times higher than for the C2C group (26% - effect size 0.85).

The B2F group started out with the lowest scores, but ended with the highest scores.

Figure 1: Raw growth in average score per student by group for pre-test and post-test, lower primary

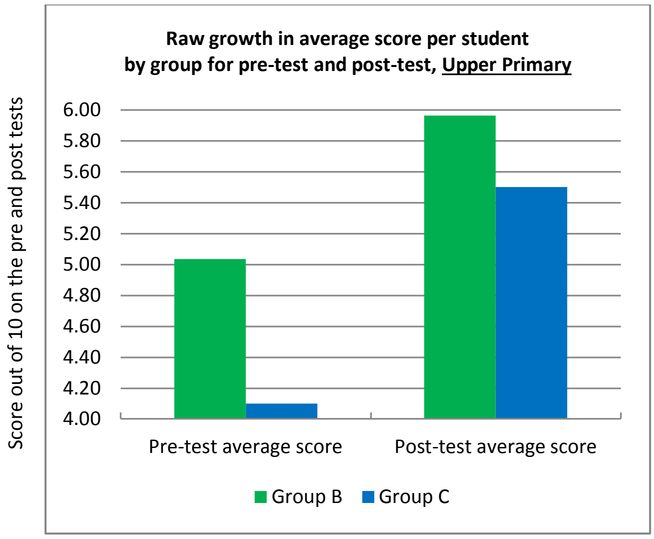


Figure 2 (left) shows the same pre- and post-test data for **upper primary** students. There was not a C2C group for upper primary, so results are only shown for Groups B and C.

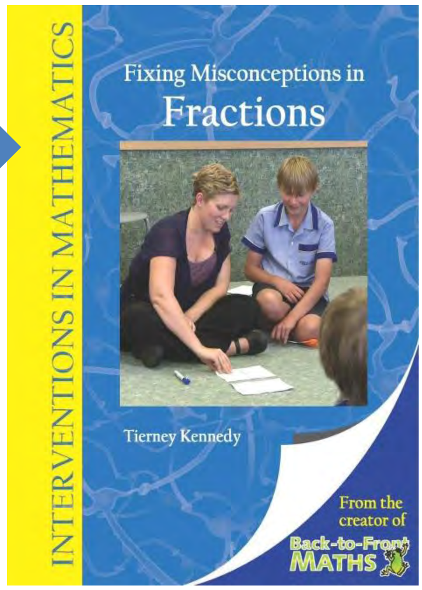
Group B showed a growth of 18%. This is a higher growth rate than the same group in lower primary, and much higher than the C2C group for lower primary. Group C showed a growth of 33% which is also higher than the same group in lower primary by approximately the same rate. Because of the wide variety of classes involved in the project, Group C started off significantly below Group B in their pre-test scores, but narrowed this gap throughout the project.

Figure 2: Raw growth in average score per student by group for pre-test and post-test, upper primary

How we made that growth happen:

In late 2013, Tierney Kennedy ran a two-day Facilitator Training Course with the numeracy coaches and key teachers for schools from a specific EQ region. During this course the participants learned how to use a four-step questioning approach to diagnose and fix student misconceptions.

For details on this four-step approach check out the extract from our new book, ***Interventions in Mathematics: Fixing Misconceptions in Fractions*** in this newsletter, or the DVD set *Teaching Back-to-Front with Tierney*.



Schools then opted to be part of the pilot project and selected their own teachers to take part.

Participants from the Facilitator Training Course trained their own staff in how to use the approach and were provided with *Back-to-Front Maths website subscriptions* and also a copy of the DVD series, *Teaching Back to Front with Tierney*, to demonstrate the teaching approach. Participating schools also received one or two days of Professional Development with Tierney, focusing on using *Back-to-Front Maths* to find out exactly where their students were stuck and choosing appropriate lessons to intervene. After this initial set-up, teachers went back to their own classes to run the programs.

A follow-up day was held mid-way through the project to answer any questions and set common goals, and I was available for Skype sessions if the teachers or coaches chose to make use of these.

> *It was a pretty simple process: train coaches to train their own staff, provide them with a model to follow and give them eight lessons to implement and see what difference it made. The most important part was done in the classrooms by the participating teachers – giving problem-based teaching a try.*

We initially only planned to have Group A (C2C) and Group B (8 lesson intervention) classes, however after observing her students' reactions to a lesson that I ran in one of the classes at her school, one principal made a quick decision to give her teachers permission to use as many B2F lessons as they wanted to and abandon C2C for the project period (thereby forming Group C).

How we developed the tests:

NAPLAN test questions are a great way of working out if students have truly understood concepts as opposed to simply memorising facts and procedures. Their non-routine nature, the high prevalence multiple-choice questions that target misconceptions and their unpredictability all ensure that conditions are not only fair, but also focus on the Understanding proficiency rather than Fluency. When we were choosing a method for measuring real improvement in student understanding, NAPLAN questions stood out as the best assessment tool.

We started by identifying specific concepts within place value and fractions from within the Australian Curriculum for both lower primary and upper primary classes to test (see side box). Questions from NAPLAN tests between 2008 and 2013 that specifically addressed these areas were identified and the percentage of students across the state that answered each question correctly was compared. These questions were then organised into a pre- and post-test, by matching pairs of questions that tested the same concept and had similar results for use in the pre- and post-test so that both tests were fair. Where enough similarity could not be found the same question was used on both the pre- and post-test.

Place value (PV) and fractions (F) areas used in lower primary tests were:

- The value of digits and expanded form (PV)
- Number lines and relative size of numbers (PV)
- Numbers before and after (PV)
- Addition and subtraction using place value to partition numbers (PV)
- Non-standard representation of unit fractions (F)
- Fractions larger than one whole (F)
- Comparing and ordering fractions (F)

Place value (PV) and fractions (F) areas used in upper primary tests were:

- Multiplying and dividing by 10 (PV)
- Number lines and relative size of numbers (PV)
- Numbers before and after (PV)
- Addition and Subtraction using place value to partition numbers (PV)
- Decimal numbers between other numbers (PV)
- Non-standard representation of unit fractions (F)
- Linear representations of fractions (F)
- Non-standard representations of simple fractions (F)
- Equivalent representations (F)
- Operations with fractions (F)

If you would like to find out more about the pre- and post-test development, including which questions were on the tests you can do so by emailing: tierney@kennedypress.com.au

Interpreting the results:

It is clear from this project that it is possible to catch students up on critical concepts such as Place Value and Fractions with a limited intervention period using problem-based teaching. The Lower Primary results for the B2F group showed approximately four times the growth in student understanding for Place Value and Fractions as the C2C group over the course of ten weeks. If these results fairly represent the growth in student understanding for C2C for the average term, the B2F students learned as much in one term as the C2C students would have learned in one year. This is consistent with our empirical experience of schools switching from direct instructional programs(including C2C and others) to Back-to-Front Maths.

While Place Value and Fractions represent only two of the concepts to be learned in primary school, they are both widely acknowledged as critical for mathematical development (National Mathematics Council, 2008, NMAP, 2008). The Middle Years Research Project found that development of deep understanding in six Big Ideas in Number during primary school was a reliable predictor of student success or failure in middle school mathematics (Siemon, Virgona, and Corneille, 2001). Place Value and Fractions represent two of these six concepts identified as Big Ideas.

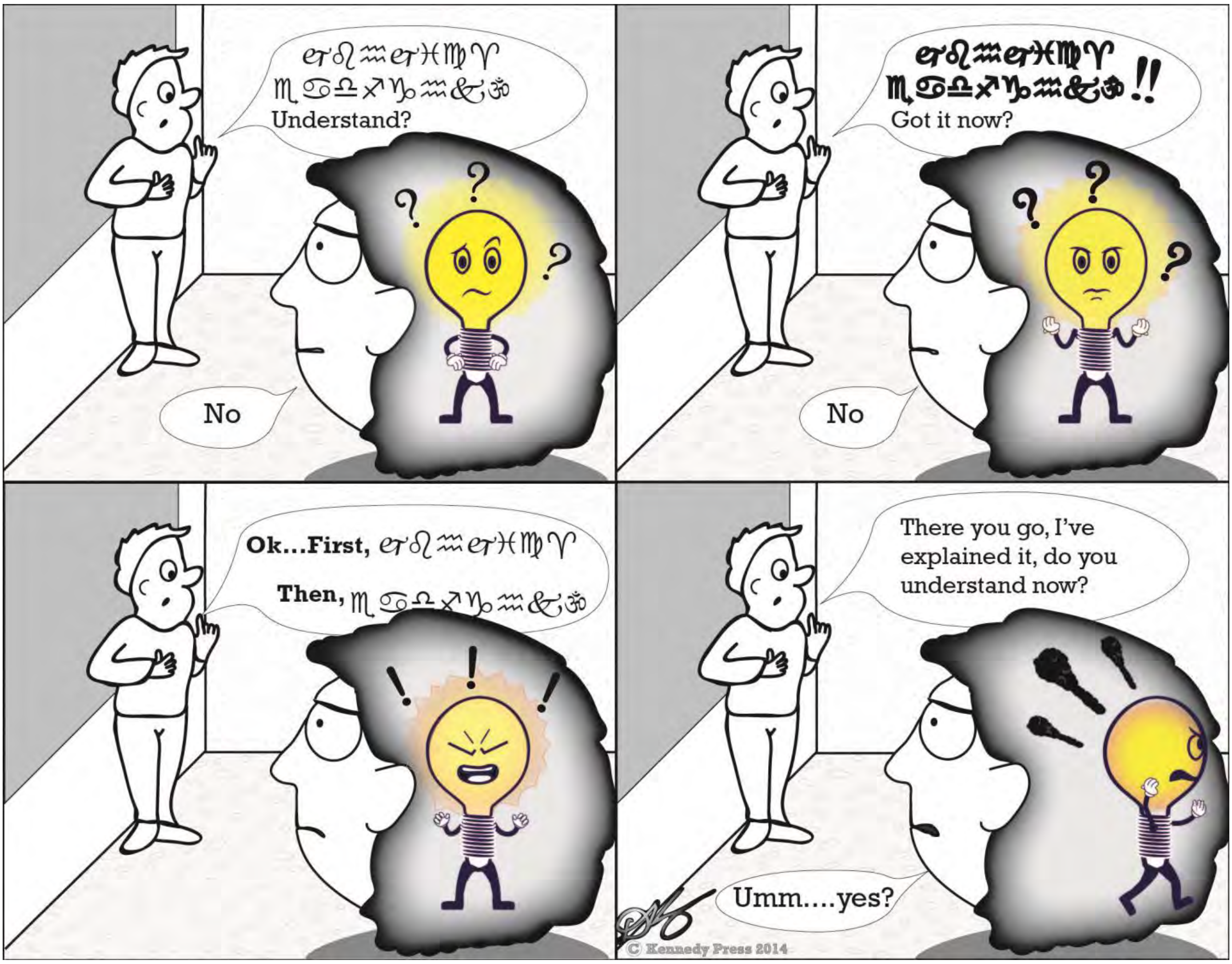
What does this mean for us?

If what we have been doing up until now hasn't worked for the kids in our classes, it is extremely unlikely that doing more of the same will fix things.

Simply put,

If we want different outcomes then we need different inputs.

Problem-based teaching has the potential to create "light-bulb moments" for students, accelerating their learning and helping them to catch up quickly on concepts that they have missed.



If you want to see the same growth for your students, here are the next steps to take:

1. Recognise the difference between memorisation and understanding. Just because a student can remember a procedure on the day does not necessarily mean that they understand the concept. To find out more about the difference between Fluency and Understanding, check out this article on [What the Proficiency Strands Mean].
2. Download these [five-minute diagnostic tasks] and implement them across your school to see if students really understand critical number concepts. Read through issues 10 and 11 of *The Insightful Classroom* to find out about critical number concepts to get right in lower, middle and upper primary.

3. Set aside a five-week period to focus on training your staff and identifying critical concepts to focus on improving. Encourage your teachers to implement problem-based teaching for three lessons per week for this period.
4. Train your staff in the four-step questioning process for diagnosing and fixing student misconceptions. Options for training include:
    a. Watching the lessons from *Teaching Back-to-Front with Tierney* DVD Set and following this with a group discussion about implementation options.
    b. Send two or three of your key staff members to a Facilitator Training Course. Term 3 options include courses in SE QLD, Central QLD and Adelaide, with possible options for other states in term 4. Please send any expressions of interest in hosting or attending courses to admin@kennedypress.com.au
    c. Book Tierney to spend a day or two with in your school, running example lessons, mentoring sessions and after-school workshops. There is limited availability for the rest of this year, so get in quick and we will see what dates are possible.
    d. Organise a Skype session with Tierney during a staff-meeting time to introduce your staff to the problem-based approach. Email tierney@kennedypress.com.au for details.
5. Resource your staff to effectively implement a problem-based approach. Resourcing options include:
    a. Web-access to the Back-to-Front Maths website for individual teachers or whole schools
    b. Teaching Resource Packs which have a copy of both student and teacher books for Back-to-Front Maths
    c. Student workbooks for 2015: three-day-per-week problem-based lessons, with free Teaching Resource Packs and website access for all staff members. NAPLAN improvements have been made in over 90% of schools using book-based resources with trained Facilitators within 12 months of implementation.
    d. Our new series, *Interventions in Mathematics* has just released its first book: ***Fixing Misconceptions in Fractions***. Use the order form on the last page of this newsletter to order your copy now.

What are your next steps?

References

National Mathematics Advisory Panel (2008). Final Report: US Department of Education

National Mathematics Curriculum: Framing paper (2008), National Curriculum Board (p.8)
Found at: http://www.ncb.org.au/verve/_resources/National_Mathematics_Curriculum_-_Framing_Paper.pdf

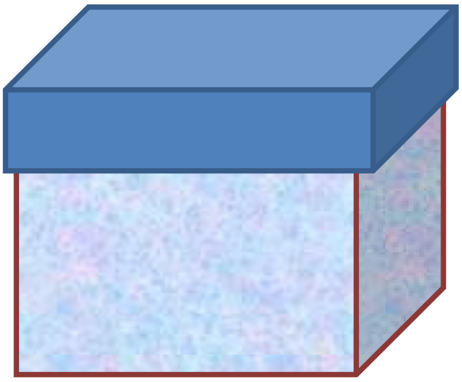
Siemon, D., Virgona, J. & Corneille, K. (2001) Final Report of Middle Years Numeracy Research Project 1999-2001, RMIT University: Melbourne

A word to HOCs

Creating an Extension Box

Tierney Kennedy - Education Consultant, Author and Editor

It can sometimes be tricky to genuinely cater for gifted kids during maths. The temptation is often to give them more and more practice questions to keep them "busy", or to just make the numbers bigger so that the calculations take more time. I'd like to give you a fairly simple, but very effective alternative… the Extension Box.



Choose two or three students to work together on a project. Place an investigation and all of the materials that you would need to do the task into a box that can sit permanently on your shelf. Subscribers to the Back to Front Maths website can download suitable tasks here: http://www.backtofrontmaths.com.au/investigations
You can also write your own or download and adapt online ones.

Once you have created your box it's time to decide the best way to use it. Personally, I find that it is best used when I need to work with most of the class on building fluency – often in the lower level skill/drill lessons when students who have already mastered that content would be bored and create behaviour problems.

During that time I ask a small group of students to get their box and work on their project. Because this is a task that is driven largely by the students, I expect them to basically look after themselves during this time. I usually set aside a 15 minute block at some point where they can come and talk to me about how they are going, ask any questions and plan whatever else needs doing. However, I make it very clear that at times when I am working with the rest of the class I expect them to work out any difficulties without my input.

Some tips:

- Always have at least two children working. One child working alone feels excluded.
- Choose tasks with a real-life application of relatively simple maths. Focus on developing and adapting patterns rather than on working with really big numbers.
- Consider having students develop their own tasks within guidelines. Work with them to design the task based on something that they find interesting.
- Investigations dealing with a set cost and a cost-per-item (e.g. a field trip with a bus hire and then an entry per student) are great ways of applying algebraic thinking, negative numbers and fractions to real life.

Tierney

In The Next Issue >>>

Resourcing your school for next year

Planning your assessment for end of year reporting

Professional development opportunities

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Alternatively, visit www.backtofrontmaths.com.au/teachers

Feedback and questions are always welcome: Contact Education Consultant Tierney Kennedy at tierney@kennedypress.com.au

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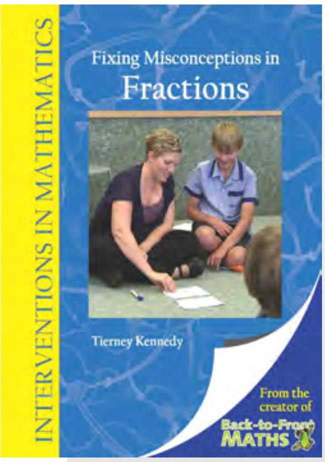


Fixing misconceptions step 1: Diagnosing problems

Why routine questions don't cut it:

Kids don't want to risk being wrong or feeling stupid. Therefore, instead of answering a question with what they really believe or saying that they are not sure, students will tend to give the "right answers" whenever they can. If we only ask routine questions, then we will only get back routine answers. Unfortunately, these routine answers can hide underlying misconceptions which fit within the domain of understanding rather than memorisation.

If we want to diagnose misconceptions, we need to ask students questions to which they cannot have simply memorised the answers.

__NAPLAN and Misconceptions:__

One widely-available source of non-routine questions that targets student misconceptions is NAPLAN tests. These questions are deliberately non-routine, yet with relatively simple content. The multiple choice questions even build common misconceptions into the possible answers!

We need to ask non-routine or "weird" questions that deliberately target students' intuitive understanding of the underlying principles, patterns and connections in maths.

The lessons within this book contain diagnostic tasks that are designed to draw out student misconceptions. At times they will seem weird or even somewhat unfair. This is deliberate - we need to go beyond routine questions so that we can find what students really believe, rather than just what they have memorised.

How to ask diagnostic questions:

It is important to be aware that kids are very good at reading our body language. They often know than an answer is right or wrong just by our face and the tone of our voice!

When trying to diagnose misconceptions, we need to make sure that we are assessing a student's real understanding, not their ability to look at our eyebrows. Try to maintain a very neutral expression and tone of voice. If we express surprise or laugh at a wrong answer from a student they will know that it is wrong and will immediately censor their thoughts. If we don't know what they *really* think then we can't do anything to fix it. For tips on how to encourage risk taking in your class so that students are not concerned about being wrong, watch our short video at: http://goo.gl/PPLXq7

What to look out for:

When using diagnostic questions the purpose is to test the fragility of student understanding. Do they still understand the concept if we ask a weird question or does it seem to just fall out of their heads? Watch out for students who are watching your face closely to check if they are right. Watch for those who wait for others to answer first. Watch for those who answer, but with the tone of a question in their voice. Once you have spotted some kids in these categories check to see if they change their minds if you change your expression or voice.

Fixing misconceptions step 2: Confronting misconceptions

Why our current approaches aren’t working:

As teachers we are pretty good at knowing when a student doesn’t understand a concept. While we might not know exactly what is going wrong, diagnosing a problem is generally much easier than fixing it. This section shows how to confront and fix misconceptions so that the kids abandon that wrong idea and grow their intuitive understanding.

From what I have observed we tend to respond in one of these two ways when we find misconceptions:

1. Tell students that they are doing it wrong, and proceed to give them detailed steps to follow to get the answer right. We help them to practice the steps until they can "remember" them. The problem is that students are left trying to memorise steps that are in conflict with their intuitive understanding of maths because they haven't *changed their own mind* about the misconception first. They still have the underlying belief that is causing the problems because it hasn’t been undone.
2. Open up the problem by asking students to "try something else" - randomly guessing until they hit on the correct response and then trying to remember that one. Again, the misconception is still there because it hasn’t been undone.

Herein lies the difficulty,

*Misconceptions are a problem with understanding, not with fluency.
Telling a student how to work out the answer and giving them steps to memorise does not fix understanding, it fixes fluency.*

If kids fundamentally don't get a concept, they need to have this problem fixed before they can successfully learn a new concept. We need to undo their wrong understanding before we can lead them to develop a new concept. Otherwise we keep chasing our tails as our students keep "forgetting" what they have "learned".

How to use logical questions to confront and undo misconceptions:

To change our intuitive understanding or our beliefs about how something works we need to have a really good reason for doing so. Someone else telling us that we are wrong is generally not a good enough reason. We need to work out for ourselves that what we believe actually doesn’t make any sense and would never work before we are ready to abandon it.

To confront a misconception we need our students to figure out for themselves that what they are thinking doesn’t make any sense and would never work. We need to take them through a logical sequence of thinking that forces them to abandon their wrong ideas - but it needs to be their own thinking rather than ours. We can’t short cut the process by trying to substitute our thinking for theirs. That doesn’t work because they won’t have actually done the thinking for themselves and therefore won’t remember it afterwards.

To effectively confront misconceptions, we need to start by narrowing the options so that a student doesn’t have too much information to try and consider, and then make a series of very small changes to the situation until they realise that the situation itself doesn’t make any sense. By using a sequence of increasingly closed questions we limit the possible responses, presenting the student with situations that challenge and confront their thinking. These questions aim to point out the illogicality of a student’s thinking. They become more and more narrow, forcing the student to reconsider their initial answer each time until they realise this illogicality for themselves. A good example of this process is included in the next few pages.

Developmental sequence of concepts

Symbols:
DE – diagnostic-exploratory lesson (explained on page 9)
PB – pattern-building lesson (explained on page 10)
GA – generalising and application lesson (explained on page 10)

Lesson 1: Are different shaped halves the same size or not?

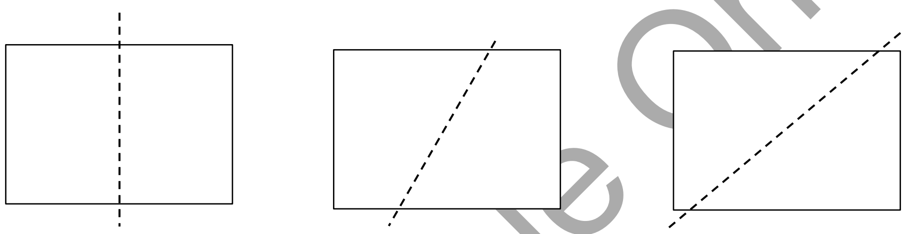
Lesson type: Diagnostic-exploratory in two parts Time Allocated: 1-1.5 hours

Concepts targeted:

- Fractions need to be “fair”. If the pieces are to be given the same name, then the size of each piece needs to be the same.
- The “whole” needs to be the same when comparing fractions. We can’t add half of a triangle to one third of a circle – we need to start from the same “whole”.

Resources:
You will need lots of A4 paper, copies of the worksheet on page 24 and a pair of scissors for this activity.
To watch this lesson in action and with explanation, see the grade 4 lesson from the *Teaching Back to Front with Tierney* DVD series.

Step 1: Diagnostic question part A
Ask students to make half of an A4 piece of paper. Then ask them to make as many different halves as they can that are still really half of the paper. If they struggle, challenge them to make the “weirdest half” that they can which is still really a half. They should realise that there are limitless ways to fold a rectangle into halves as long as the fold line goes through the mid-point of the rectangle:



<u>Watch out for the following misconceptions:</u>

- Big halves and little halves
- Only symmetrical shapes can be half (e.g. you couldn’t fold the paper through the corners diagonally because everything wouldn’t line up nicely). These kids often cut the rectangle into a square and then cut the square diagonally, making three pieces.

Step 2: Confrontational questions for part A if needed

1. Cut along the fold and hold up both pieces. Ask the students if this was a really yummy cake which piece they would want to eat.

2. Ask if the pieces are halves. Ask why they think the pieces are halves or not halves. Hopefully this will establish the concept that halves have to be fair. Watch out for the term “even” as it is confusing to kids. “Fair” is a much better word to use than “even” as kids confuse the usage of the terms odd and even with that of equivalence.

3. Sometimes kids will tell you at this point that the pieces are halves because there are two of them rather than mentioning their size. If so, cut a really small corner off another piece and hand it to a student saying, “Ok then this can be your half and this can be my half. What do you think now?”. Usually the students will respond that the pieces aren’t fair. Ask if they think that matters or if they can still both be called halves if one is bigger.

4. Sometimes students decide that the pieces are not fair but that they are both still halves because half means two. I have actually had a student say, “I know it doesn’t make any sense Miss, but that is what they are called”. At this point try asking, “So we don’t think that these pieces are at all fair, but we still think that they should be called halves? Does that seem right to you?”. Usually students will respond that it doesn’t make much sense. I usually tell them that maths should make sense.

If something doesn't make sense, then it is usually wrong. Then I ask, "Does it make sense that things of totally different sizes would be given the same name, that both would be called halves?". Usually they say that it doesn't make sense. I say, "Well if it doesn't make sense, then it can't be right. These are going to go in my ***not halves*** category". I then create two categories on the board: halves and not halves. We test each idea for fairness and put each onto the board using one of those two categories.

5. For kids who cut the rectangle into a different shape first, get them to bring out all of the bits that they cut. Ask them to pick half. Then pick up the rest of the bits and say, "Ok you can have that half and I'll have the rest. What do you think?". Usually they realise that it isn't fair because they have cut off part of the paper first, then they go back to try again.

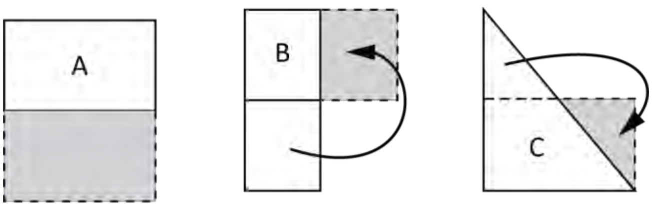
Step 1: Diagnostic question - part B (you can use the worksheet provided on page 24 for this question)
Label each of the differently shaped halves with a letter (A, B, C, D, E etc.) You will need to have at least four differently shaped halves, hopefully more. Ask the kids to write down the letter or letters for which of the pieces they think is the biggest. The pieces are actually the same size, but as this is a diagnostic question it is designed to show misconceptions. Typically, one of the students will ask if they can pick more than one shape. I usually respond with, "Sure – you can pick one, two, three, however many you want". I then check each student's answer individually by walking around and checking the letters that they have written. If they have answered with, "all of them" then I look at their faces – are they asking if that is right or are they really sure? If they aren't really sure then I double check their understanding by asking, "Ok, do you want a second guess in case you're wrong?" or, "Ok, but if they're not all the same then which one do you really think is the biggest?".

<u>Watch out for the following misconceptions:</u>

- Some kids believe that all halves have to be fair, but also believe that one or more of the halves you have identified are still bigger than the others. Students in this category are often missing *conservation of area* because they have not understood the two-dimensional array model of multiplication. An example of this thinking can be seen in the photograph on the right. This photograph shows 12 and 13 year old students voting that a particular half was bigger than the others even after they had gone through the initial testing phase of the first diagnostic question. This is a very persistent misconception, so make sure that you spend the necessary time to fix it properly using the questions below in step 2.

Step 2: Confrontational questions for part B

1. Hold up two of the halves in your hands. Ask the kids if there is a way that you could test to see if they really were the same size or if one was bigger. Typically, they will automatically try lining the shapes up and seeing if they can cut off the part that is overlapping and move it around to make both shapes the same (see below for process). If not, try holding the shapes together so that the dimensions align (place two sides of the same length together), then hold this up to the light so that the kids can see which parts line up and which parts don't. Usually this is enough to help them get the idea. Demonstrate cutting up the different shaped halves to overlay them if needed, making them all into the same shape as half A. Use this to check student answers.



2. Get the kids to check each shape and ask, "So which is bigger now?". Hopefully they will realise that each is actually the same size.

3. Double check that their understanding really has changed by moving the cut pieces back into their original position and asking, "So which is bigger now?". Hopefully they will realise that each is actually still the same size, but there are usually some kids who think that the size has changed again and one is bigger.

4. Repeat the process of moving the pieces, making tiny movements and asking, "How about now?" until the pieces are back into the same as shape A. Usually the response is, "bigger, bigger, bigger… now they are the same". Make a tiny movement to shift the two pieces apart again and ask, "How about now?". See if the student thinks that it is bigger/smaller again or if they are still the same. Hopefully just making tiny movements to isolate the spot where the shape "becomes bigger" will be enough to change their minds. If not, move on to step 5.

5. Cut shape A to be the same as shape B. Now move the pieces of shape A to look like shape B, keeping shape B's pieces in the position of shape A. Ask again, "How about now?". Often at this point a student says, "Now this is confusing – I don't know which one is bigger!". If so, you can respond with, "Well, is one actually bigger or does it just look bigger? Which one has more cake if you could eat both pieces?". If not, try step 6.

6. Swap one of the pieces of shape A with an identical piece from shape B. Ask again, "How about now?". Keep swapping the pieces and moving them back into the other positions until the student responds by saying either that they are the same or that they don't know. Question as per step 5 above. If they don't do this, you will need to go back to an intervention on drawing arrays for multiplication facts (e.g. draw 3 x 4 as three rows of four, then rotate the drawing and see if it is still the same). Please note: As this is a separate concept from fractions it has not been included within this program. Information and lessons on how to fix multiplicative thinking can be found on the Back-to-Front Maths website. A videoed lesson showing how this misconception was fixed in a grade six class can also be found in the DVD series, *Teaching Back to Front with Tierney*.

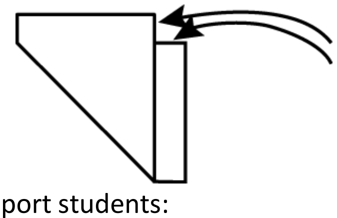
Step 3: Leading questions

- "Do you think I can have a big half and a small half or do they have to be **fair**?".
- "Is it a different amount of cake, or does it just look different? Can it look different but still be **fair**?".
- "What have you changed your mind about today? What have you learned?".

Step 4: Generalising questions:

- "Do you think that halves always have to be fair? So, what if they look different but are the same size really - would that still be ok?".
- "Ok, let's try making the weirdest shaped half that we can from this A4 paper, but it still has to be fair".
- "How about for other fractions? Do they have to be fair too or is it just for halves?".
- Reiterate: "So, we have learned today that fractions have to be fair".

<u>Differentiation:</u>
Extension students: Have them work out other weird ways to fold the paper to still make half. One interesting principle that a *grade one* student came up with is this:



As long as these two distances are the same it will always be one half!

Support students:

1. Reinforce the concept by cutting halves from other shapes.
2. Draw arrays of simple multiplication facts (e.g. 2 x 3) and then swivel them by 90° to see that it is the same amount. Repeat with lots of different grids or arrays (e.g. 6 x 5 = 5 x 6), then go back to fractions.