Journal problem 4: Visualising tenths

Complete Blast activity A12 before this problem.

Introduction:
This focuses on helping students to build connections between what they have already learned about tenths. While it is theoretically not a difficult problem watch yourself for leading students too heavily at the start. It is very important to see what they really believe about decimal numbers and help them to change their own minds rather than us just telling them. Make sure that you read through the section on misconceptions before starting.

Write the number 23 on the board. Ask students if they can make the number 23 out of MAB in as many different ways as they can. Then mix the blocks up, and ask if it is still 23. Repeat this process several times to reinforce the idea that 23 can be moved around, stood on its end, spread out etc. and it will still be 23. Then change the number on the board to 23.7 and ask if they can make it out of MAB now. Give them tens and ones to use. Then watch out for misconceptions! They usually take two forms: (1) decimal points just separate numbers that are as big as each other (you will see them build 0.7 using 7 ones or 7 tens blocks with a dot or a big space between them); (2) decimals are "tiny" ones that have nothing to do with base 10 and you can make them by cutting the one into various sizes (halves, quarters, eighths and sevenths usually).

Some students (not usually many) will realise right away that it can't be done. Say to these students, "well, what if I said that you were allowed to cut up the blocks? Could you do it then? You can't really cut them, but can you draw what you would make?" Watch out because most students cut the block into a weird number of parts that isn't 10!

You will need: substantial supplies of MAB (tens and ones) for this activity. One way to accommodate this is by completing the activity in 'rotational groups' time.

Leading questions and how to challenge student misconceptions without saying "no you're wrong":

- Push all of the blocks back together and ask if it is still 23.7. Then separate the 23 blocks from the 7 and ask if it is 23.7 now. Lots of students usually answer *yes*. You will need to repeat the demonstration from the introduction using just 23. If they keep thinking that separating 23 from 7 makes it 23.7 then hold up your hands and ask how many fingers you have. Put one MAB on your head (to be the dot) and hold your hands apart. Ask how many fingers you have now. Watch out

Misconceptions to watch out for:

(1) Decimals just separate numbers of the same size.
Students who think this often represent the 0.7 in 23.7 using:

- 7 more ones with a gap in between the 23 and the 7 (or a dot, or a block used to mean a dot).
- 7 tens (because it sounds like tenths).

(2) Decimals and all fractions are made by cutting in twos.
Students who think this often represent the 0.7 in 23.7 using:

- Half a block (and we need 7 of them), which when you draw it on the board and then "glue" back together makes 3.5 blocks extra to put with the 23.
- 7 quarters (using 2 blocks with one quarter of one block missing) or 3 quarters.
- 4 quarters from one block and 3 thirds from another block.
- 7 eighths of a block.
- 7 sevenths of a block (they think that .7 means sevenths) or 1 seventh of a block.

(3) Decimals mean numbers less than zero (negative numbers)

- Be aware that many students think that numbers after the decimal point are less than zero, not between whole numbers.

because lots of students say 5.5! You can then use this to challenge the idea (*Really? Did I chop one in half?*)

- Make 23 from MAB and move slowly to put just one more MAB with it. Ask if this is ever going to work (they should answer, *no that's 24*). Ask if they think that the "decimals" are going to be as big as this one. They should decide at this point that they will need to be smaller. Then prompt them saying, *"What if you could cut the MAB into as many pieces as you wanted to make the point seven? Could you draw what you would do?*"
- Draw each of the "cutting the MAB" ideas on the board in 3D. Then "glue" them back together by rubbing out the lines. Ask "*is it still 23.7?*" At this point students can exclude any ideas that use a whole block or more than a whole block.
- Focus on the patterns between the MAB using this line of questioning once you get to the point where students are debating between representations such as 3/4, 1/7, 7/8 and 7/10. You will need a 1, 10, 100 and 1000 MAB. It is also helpful to have 10 x 1000 blocks to build a 10 000 stack.
    - Hold a one MAB in one of your hands and a ten MAB in the other. Ask: "What is this block called (*one*), and what's this one (*ten*)? How many of these (*ones*) would I use to build this (*ten*)?"
    - Repeat using a ten and a hundred, then a hundred and a thousand.
    - Hold a thousand MAB. Ask what the next number is called (*ten thousand*). Ask how many thousands you would need to build a ten thousand. Build it.
    - Now work backwards from ten thousand, asking "If I only had this block (*larger one, e.g. ten thousand*), how could I use it to make this one (*small block, e.g. thousand*)?" The kids should tell you "Cut it in ten bits". Repeat all the way back to one MAB.
    - Then hold one MAB in one hand, and ask, "So how would I make the next one?" Stop before they tell you and ask if they want to change their minds on how to draw 0.7.

Teaching Tips:

- Support students: See the misconception section. Consider repeating the regrouping activity from Journals problem 1.
- Extension: Watch out for students who firmly understand what 23.7 means straight away and need extension. Have them go straight to the manipulation problem which requires regrouping across the decimal point (e.g. 2 tens, 2 ones and 17 tenths). Ask them to make 23.7 in as many different ways as they can.
- Consider using packets of straws and asking students to show you 23.7 straws. You will probably see the same misconceptions! You can also give students a loaf of bread or play-dough and ask them to show you 0.7 of a loaf.
- Students often think that the decimal point is a full-stop and that numbers cannot be regrouped over the decimal point.
- Try using photocopies of MAB and cutting them up to make tenths and hundredths (e.g. a $1cm^2$ square sliced into 10 or 100 slivers).

Follow up ideas:

Complete Blast activities A13-A16.

Problem 4: Visualising tenths

TR p56

What does it look like to make the number 23 using MAB?
Now, what does it look like to make 23.7 using MAB?

Teacher-led warm up task: make the number 23 out of MAB. Is there another way that you could make the same number using different blocks? Draw all the ways that you can think of:

Does it matter which order the blocks are in or can we move them around and still have 23?

Problem Solving and Communication:
Now let's try making the number 23.7 using MAB. You will probably have to try lots of different ideas to find one that works, so draw each of your ideas here as you think of them. Number your ideas so that your teacher can work out how you thought about it. There is more space on the next page.

Feeling stuck?

Imagine if you were allowed to take a saw and cut up the MAB to be any size and shape that you wanted. Can you draw what you would try?

Understanding:
Check your ideas: If you push all of your MAB back together again does that still make 23.7? If not, then you will need to try something else.

More space to try out your new ideas:

You need to check with your teacher before finishing this problem.

Check your communicating:
Make sure that your drawings clearly show how you made the numbers.
If you need to, add captions or describe here how you did it:

Understanding:
Most students change their minds at least three times. Explain how your ideas have changed and how you can be sure that what you have come up with really represents 23.7:

Manipulation problems:
If you actually had those blocks to use, but you could only use 2 ones blocks, how could you still make 23.7?

Teacher initials:

Date:

Problem solving / T&R:
- o Problem solved with minimal or non-mathematical prompting
- o Some leading questions were used to prompt thinking
- o Solved after explanation
- o Did not work out solution
- o N/A- not a novel problem

Reasoning / Comm.:
(verbal, written, working and equations, or visual representations)
- o Clearly and logically reasoned
- o Easily understood
- o Understood with some interpretation needed
- o Some gaps but on topic
- o Minimal or off topic

Understanding / Reflect:
- o Connected manipulation problems to previous questions and answered easily
- o Connected manipulation problems to previous questions with some prompting, and answered correctly
- o Answered once the similarities to previous questions had been pointed out
- o Had some problems in answers but was on the right track
- o Did not answer appropriately

- o Student not observed