Workshop notes: Introducing Negative Numbers

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Negative numbers are a particularly difficult concept for many students to understand. With the Australian Curriculum introducing negative numbers in earlier years it is pivotal to develop deep understanding from the start. In this workshop we will deal with the following concepts:

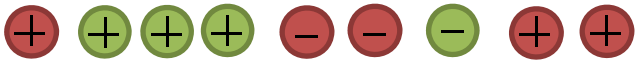
1. Negative numbers have a quantity
2. Negative numbers are smaller than zero
3. Negative numbers can be used in operations without needing lots of rules (e.g. double negatives)

Negative numbers have a quantity: The circle swap game

This activity can also be useful for introducing the concept of grouping like terms.

Circle Swap Game:
Each student is given a handful of circles from each colour. These circles have a positive sign on one side and a negative sign on the other side. You then write a sentence on the board that tells the students what to make using their circles (see the example below). They combine circles of the same colour, and can remove a pair of circles (one with a + and one with a –) of the same colour. Pairs like these cancel each other out and can be taken away. Once any circles that can cancel have been removed, students simply work out how many circles of each colour they have.

For example:
The teacher might write: +1red +3green -2red -1green +2red
This looks like:



The students would then cancel out any  with  and any  with 

They would be left with:

Which is written as: +1red +2green

Once students have the hang of the game, it is very simple to say, “Do you mind if I just write r instead of red and g instead of green?” Plan a few rounds, then end up with negative numbers and can see that they have a value (e.g. -2r -3g). I often like to give students a very long and involved equation and then say “any reds are worth $1 and any greens are worth $2. Do I owe you money, or do you owe me money?”

This game is a very simple way of associating a quantity with negative numbers. Negative apples and bananas are much harder to imagine!

Negative numbers are smaller than zero

This lesson is taken from Journal problem 3, year 7, *Back-to-Front Maths* (page 9-10) and the lesson plan from page 56 of the year 7 Teaching Resource Book

Problem 3: Using number lines with negative numbers

Problem Solving:

Stretch a piece of string across your classroom. Label one end of the string -$50 and the other end +$50. Use this to work out the following problems. When you have worked each out, use your calculator to help you work out an equation for each one (e.g. -20 – 10 = -30)

Warm up: Write the equation and the answer.

1. Starting with a bank balance of +$10, I spend $20. What is my bank balance now?
2. Starting with a bank balance of +$10, I deposit $20. What is my bank balance now?
3. Starting with a bank balance of -$10, I spend $20. What is my bank balance now?
4. Starting with a bank balance of -$10, I deposit $20. What is my bank balance now?

Understanding and Reasoning:

How did you come up with your solutions? What process did you use to solve the problems? Explain the patterns for adding and subtracting with negative numbers.

Problem Solving:

These situations are much trickier. Your teacher will give you the answers if you need them so that you can work backwards to figure them out.

5. Starting with a bank balance of $10, I do the opposite of depositing $20.
6. Starting with a bank balance of $10, I do the opposite of spending $20.
7. Starting with a bank balance of -$10, I do the opposite of depositing $20.
8. Starting with a bank balance of -$10, I do the opposite of spending $20.
9. Model these equations using the number lines below each one. Write a sentence to explain what they mean in a similar way to those in the questions above.
    a. -20 – 10 =
    b. -20 – -10 =
    c. -20 – +10 =

Lesson Plan for Journal problem 3 above:

Leading questions:

- Modelling for the whole class using the number line: ask students to find $10 on their number line. Ask students if this is money that you have or owe (you have it). Place the card on the line with the black $ sign facing upwards. Then read out the first question (start with $10, spend $20). Ask students to replace the $ sign at the $10 and work out what to do with it when you spend $20. Move onto the rest of the questions when you are sure that they know which direction to move the $ sign in when they spend money (towards the negative end) and when they deposit money (towards the positive end).
- Where do you think the zero would go? How about the 50? Where is the $10 that I have? Where would I put $20 that I owe? Let's flip the card over to be "in the red" when we owe money. Which number is the point where we flip the card?
- So if we have a balance of $10 in a bank account and then we use a credit card to spend $20, what does that mean? Do we owe money to the bank? How much?
- What would it look like if we started off owing money? How about if we owed $10? Now what would happen if we spent more money? Would we owe more or less to the bank? Show me using the number line.
- What does it mean when the question asks you to "do the opposite of spending" or "do the opposite of depositing"? Tell me about the operations involved and how they change.
- How do the equations relate to "do the opposite of" situations? What operations are involved? Explain to me how they work.

Misconceptions to watch out for:

- Students who try to move in the wrong direction (e.g. if we owe $20 then spend $20 they say that we owe nothing because they do 20-20=0).
- Students who do not realise that each $10 is the same distance apart.
- Students who think that decimal numbers are less than zero so try to give you decimal numbers as answers. Decimal numbers describe parts of wholes, not amounts less than wholes (e.g. 0.5 is between 0 and 1 - it is not 5 less than 0).

Negative numbers can be used in operations without needing lots of rules:

The commonly used idea of "two negative signs join together to form a positive sign" creates a lot of student misconceptions! Check out the box on the right. Try using the questions below to challenge student thinking, using the number line from the previous activity.

Thinking questions from the grade 7 Teaching Resource book for *Back-to-Front Maths*

1. If I have a bank account and I spend money, am I adding to or subtracting money from the initial balance? How do you know?
2. My bank account is positive. Am I adding to or subtracting money from the initial balance when I spend money?
3. My bank account is positive. Am I adding to or subtracting money from the initial balance when I deposit money?
4. My bank account is negative. Am I adding to or subtracting money from the initial balance when I spend money?
5. My bank account is negative. Am I adding to or subtracting money from the initial balance when I deposit money?
6. Does my bank balance being negative or positive change whether spending money is adding or subtracting? Explain your answer:
7. Does my bank balance being negative or positive change whether depositing money is adding or subtracting? Explain your answer.

Misconceptions to watch out for:

- Start with a negative number then subtract something means a positive number (e.g. start with a negative bank balance and spend money to end up with a positive balance!)
- If two negatives join to make a positive, then a negative and a positive join to make a star, which means that you multiply the numbers

I like using "gap" questions with starting and ending points, so that students need to work out what operation was used to get there:

1. Start at \$20 and end at \$10. What happened? Write it as an equation. Repeat from \$10 to \$0, from \$0 to -\$10, from -\$10 to -\$20
2. Start at \$10 and end at \$20, what happened? Write it as an equation. Repeat from \$0 to \$10, from -\$10 to \$0, from -\$20 to -\$10
3. Write -10 - ☐ = 0. This will help the kids work out that they need to subtract a negative number to work it out.

Some equivalence questions from Journal problem 15, grade 7 *Back-to-Front Maths*

$6 - 2 = 1 - \square$

$3 + 4 = 9 + \square$

$10 + 3 = 12 + \square + \square$

$10 + \square = 5 + \square$

$2 \times (4 + \square) = 9 - \square$

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