What is Reasoning in its simplest form?

The explanation below is my simplified version of what it means to Communicate or Reason. This is only meant to provide a starting point for discussion.

Reasoning involves the **demonstration of the mathematical process** that a student has gone through to solve a problem. This may include their working or equations, an oral explanation or set of questions and answers, a physical demonstration using materials, a diagram or model, or a written explanation etc. Mathematical communicating is about how clear and accurate the mathematical process is, not about literacy skills.

Some questions to consider when examining communicating include:

- What process have they used? Is this process mathematically valid? Have they shown enough of their process to be able to follow it?
- How well structured is their process? Is it logical and well-reasoned? Does it give enough detail? How much do I need to interpret or "read into" this process?

Useful questions for consideration when grading Reasoning:

1. Is the student's process, as they have shown it, mathematically valid (will it generate the right answer every time)?

2. Is the student's process relatively easy to follow, or does it require interpretation?

3. It the student's process detailed and logically structured such that someone exactly following the steps demonstrated or explained would get the same answer?

What is Understanding in its simplest form?

The explanation below is my simplified version of what it means to Understand. This is only meant to provide a starting point for discussion.

Understanding is about **generalising mathematical principles** or patterns that underpin a problem, and **making connections** by adapting, extending and applying these principles to a range of situations.

Some questions to consider when examining Understanding include:

- Have they clearly shown an understanding of the mathematical principle or pattern in the problem? Can they use this pattern or principle to solve a similar problem?
- Have they made connections to another situation by:
  - adapting the pattern (e.g. altering their formula or equation to account for a change in circumstances)?
  - extending their use of the pattern (e.g. by solving a non-standard question or one that involves a different use of the pattern)?
  - applying the pattern (e.g. coming up with multiple solutions to an open-ended or real-life problem)?

Useful questions for consideration when grading Understanding:

1. Is the student's principle or pattern mathematically valid (will it always work even when the numbers are different)? Can this same pattern be applied to solve a similar question?

2. When asked a question that required the student to make connections, did the student adapt his/her pattern to account for a change in circumstances or did he/she tend to view each change as a whole new problem and go back to the start each time? Did he/she make links between the *problem-solving* that he/she had already completed and the new problem being asked?

3. How perceptive was the student when making connections? Did the student adapt his/her pattern easily in a variety of circumstances by making his/her own connections? Did he/she come up with many solutions to open-ended problems or just a few? Was the student flexible when manipulating their equation or formula?

Reasoning Tips and Strategies:

Questions to elicit Reasoning:

- How did you get your answer/s?
- What operations did you use? With which numbers?
- What did you do first? Then what did you do after that?
- How do you know that you're right? How do you know that the answer wasn't ____ instead? Prove it to me.
- What do you mean in this part? Explain it to me.

Common problems and some useful tips:

1. When asked how he/she came up with their solution student answers, "I just knew it".
    1. Change the numbers in the question and ask them the steps for solving the new question.
    - Ask the student to tell you how to work out the answer to the new question without telling you the answer. "As soon as you tell me the answer then I won't be able to mark it. I want to try and follow your steps and see if I can get the answer without you telling it to me." Scribe for the student as needed.

2. Student had the answer "pop" into his/her head and has no idea how to prove it, or what thinking they went through to get there.
    - Ask the student to "help" someone who is stuck. Watch what they do and scribe their talking.
    - Deliberately make a mistake or give the wrong answer and have the student correct you. "So if I… does that work? What do you think? Prove it."
    - Give the student other similar questions to solve and ask them to show you what is the same about each of them. "How is this question kind of the same as this other question? What is similar? What do you have to do to solve each of them?"

3. Student has trouble showing all of the steps and tends to skip bits.
    - Give the student sentence starters or equation starters with parts to complete. You might even provide words, phrases or numbers on sticky labels to stick into the spaces. Make sure that you provide lots of wrong ones as well so that they can't just complete the sentences with the only words that fit!
    - Have the student write the "calculator buttons to press" onto boxes. Show a calculator at the side so that students can only select from the numbers, operations and signs on the calculator.
    - Give the student new numbers and go through their process, skipping all of the bits that they skip. Give him/her a chance to correct you when you make mistakes and then go back and add those parts into his/her own process.

'Understanding' Tips and Strategies:

Questions to elicit Understanding:

- What pattern/principle/procedure/equation/formula have you found?
- What is the same about all of these questions?
- How do you know that you have all the answers?
- Is there a different way to work out the answers?
- What did you work out? What is the most important thing that you found?

Some useful tips:

Students don't necessarily need to brainstorm other uses for a mathematical principle, but to make connections to new uses. Consider using a new situation and seeing if they can make connections to it instead of having them come up with their own new situations. Examples are included below.

1. Change the numbers in the question and ask them to *adapt* what they have done to solve the new problem. Look to see that they use what they have already done to work out the answer to the new problem rather than going all the way back to the start every time. For example:
   a) 26 students went to the zoo. The admission was $5 each. How much did it cost?
   b) What if two teachers went with them? What would it cost now? (do they start from scratch
   c) What if three students were away that day and missed out? What would it cost now?
   d) What if the admission price was $7 instead of $5? What would it cost for each of the scenarios above?
   e) What if the admission price was $7.50? What would it cost for each of the scenarios above?
   f) What if they also had to pay $1 each for a bus? What would it cost for each of the scenarios above?
2. Ask them a *non-standard problem* with a similar principle. For example:
   a) 2 ☐ x 5 = 115. What goes in the box?
   b) The total admission cost for a class to go to the zoo was $115. 23 students went. How much did they each pay? (non standard multiplication and division)
   c) Which of the following shapes is a hexagon (give a regular octagon, pentagon and an irregular hexagon)? (non standard shapes)
   d) What number has 23 ones and 42 tens? (non standard place value)
   e) If 1/3 of the group was 5 students, how many students were there? (working backwards, non standard fractions)
   f) I need to be at school at 3:00. It takes 45 minutes. I need to set my alarm for 5 minutes before I have to leave. What time should my alarm be set for? (working backwards, multistep)
   g) Change 1.23 cm into m. Change 1498m into mm. (non standard conversions)
3. Ask them a problem with *multiple answers* or an open-ended problem.
4. Ask them a problem with *missing information*, and have them ask you for the missing information rather than solving the problem.
5. Give them a number of *wrong answers* to a problem and ask them which one is right. Make sure that you include common misconceptions in your answers.