Multiplicative thinking and the link to quadratics:

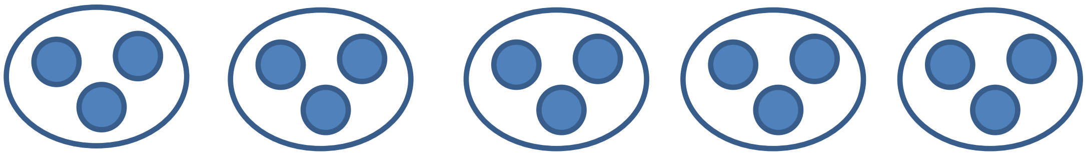
Multiplicative thinking has been identified by the Middle Years Research Project (Siemon et al) as one of the big ideas in number. Essentially it refers to the ability of a student to picture multiplication in a two-dimensional structure (array) rather than in a single dimension (additive models: skip counting, groups of).

In my own work with approximately 3000 primary teachers, I have found some important trends:

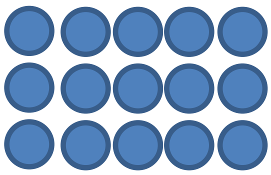
1. Teachers who picture 3 x 5 as three groups of five rather than 3 rows of 5 in a 2D array tend to:
    a. struggle when multiplying larger numbers without using the standard algorithm
    b. classify their experience of high school maths as largely "memorisation without understanding"
    c. classify themselves as "not good at maths", and assume that they were "born that way" rather than seeing their knowledge as a product of their experiences
2. Teachers who picture 3 x 5 as an array tend to:
    a. be able to multiply larger numbers without using the standard algorithm
    b. classify themselves as having been "good at maths" at school
    c. classify mathematics at school as a process of building on each concept, rather than of memorising without understanding.

Teaching methods for multiplication:

- Skip counting: counting in 2s, 3s, 4s etc. This is usually achieved by rote counting, or looking at patterns of addition.
- Groups of: Representing multiplication visually as groups. This is another way of linking addition or counting with multiplication.

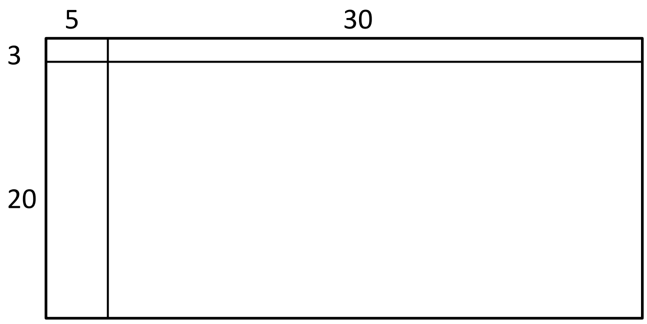


- Arrays: Representing multiplication visually in aligned rows and columns or grid structures.



Arrays are the most useful teaching method for developing multiplicative thinking.

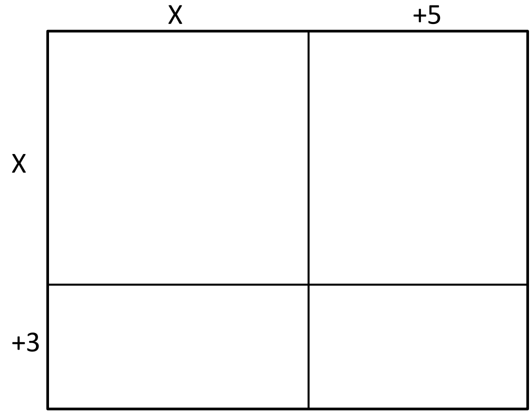
<u>Extending arrays to two-digit multiplication</u>: Example – 35 x 23



Adults and kids who struggle with two-digit multiplication have often failed to extend arrays to larger numbers, or have not developed arrays as a visual image for multiplication in the first place. When multiplying two digit numbers they tend to only do two parts: 3x5 and 20x30, instead of the four parts that there are. Visualisation of an array stops any need for memorised procedures.

Extending two-digit multiplication to quadratics: (x +3) (x +5)

Using arrays to extend multiplication to quadratics helps both adults and kids to visualise what the distributive law looks like. No memorised procedures are necessary if they understand what a two-dimensional structure for multiplication looks like.

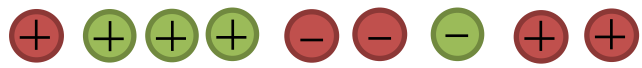


What's with all those apples and bananas?

Most students have difficulty understanding that in algebra you cannot add a to b and get ab, even when expressed as "apples" and "bananas". Combining like terms is a very important concept that once grasped makes most of algebra much simpler. I like to introduce the concept of combining like terms with a game that I call *Circle Swap*. For this game you need to have a number of cut out coloured circles for students to use. Each circle should have a + written on one side and a – written on the reverse side.

Each student is given a handful of circles from each colour. You then write a sentence on the board that tells the students what to make using their circles (see the example below). They combine circles of the same colour, and can remove a pair of circles (one with a + and one with a –) of the same colour. Pairs like these cancel each other out and can be taken away. Once any circles that can cancel have been removed, students simply work out how many circles of each colour they have.

For example:
The teacher might write: +1red +3green -2red -1green +2red
This looks like:



The students would then cancel out any  with  and any  with 

They would be left with:

Which is written as: +1red +2green

Once students have the hang of the game, it is very simple to say, "Do you mind if I just write r instead of red and g instead of green?" Once they are confident with this, change the letters to something else (e.g. put a y instead of an r). When students ask what y means say, "Yellow?" and then let them know that it is ok to use a different colour if they don't have any yellow. You can then progress to w (white) and then try for an x. When students ask what x means say, "I don't know, some colour starting with X. How about we make up a colour called Xavier? Is that ok?"

One of the advantages of the *Circle Swap* game is that students can end up with negative numbers and can see that they have a value. I often like to give students a very long and involved equation and then say "any reds are worth \$1 and any greens are worth \$2. Do I owe you money, or do you owe me money?" Negative amounts of money (e.g. credit cards) are also fun to work with.