Work Program for B2FMaths@Home Week 3 Year 6

Contents – click on the link below

How to use this work program

Accessing the online resources

To access the online resources, please go to: https://www.backtofrontmaths.com.au/b2fmathshome

Running the program each week

Each week is designed with five maths lessons so that you can do it each day. Different days have different types of lessons to make sure that students experience the kind of thinking that they need to continue growing in maths. The types of lessons include:

- **At-home investigation:** This is a hands-on task where students explore a new idea before they are taught that skill. They need to come up with an idea to try to solve the problem, try out their idea, decide if it worked or not, try again if needed, and explain what they did. If your child has time with your teacher with a webcam, the teacher will generally be doing this lesson with your child. This is the lesson that will require the heaviest input from you to help your child think through an idea and generally requires the use of some hands-on materials that are listed in the information page.
- **Connecting lesson:** This type of lesson has questions that lead students to develop their ideas and learn a new skill. It should be fairly easy for a student to do, but you will need to be available to read the question to your child as needed, encourage them to think further, and make sure that they complete the work. Most of these lessons will include 10 minutes of practising number operations or concepts through activities or games.
- **Interleaved practise lesson:** This type of lesson provides 8-10 questions from different areas of maths so that students practise remembering what they have previously been taught. Some of the questions may not be easy for your child, so feel free to help whenever you see them struggling.
- **Generalising lesson:** This lesson contains some extension material for use if your child found the week's lessons too easy. *If you would prefer*, you can spend this lesson playing more of the number games that are included in the connecting lesson or giving your child time to complete any of the lessons that they have not yet done.

Getting help

The website above will have answers to frequently asked questions as well as videos to help you successfully teach your child at home. If you have further questions or need support, please contact your child's teacher directly using the contact details that they have provided to you. If they can't answer your questions, they will contact the B2FMaths@Home team directly to get an answer within 3 days.

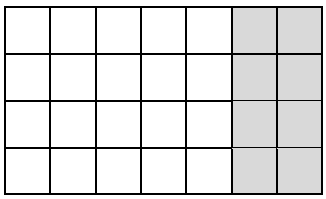
What you need to know this week

Week overview

This week we are teaching the concept arrays and counting patterns. We will particularly be focusing on arrays of objects arranged into a grid-like pattern (e.g. tiles or the top of Lego blocks). This model for multiplication and division has links with many other concepts in later years, such as area, volume, fractions and helps develop a firm foundation for understanding algebra.

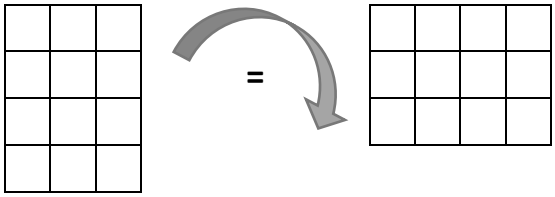
Students need to understand:

- **Distributive Property:** Arrays can be easily split to make multiplying large numbers easier.



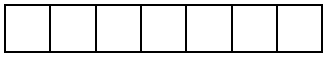
4 sevens is the same as
4 fives *and* 4 twos
4x7 = 4x5 + 4x2

- **Commutative Property:** Arrays can be rotated to demonstrate that 4x3 = 3x4.

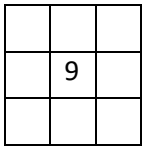


4 threes = 3 fours

- **Factors** are the length and the width of an array. The **multiple** is the amount altogether.
- **Prime numbers**, like 7, can only be made by multiplying 1 by itself. That means that they only have 2 factors: themselves and 1. When we arrange prime numbers into arrays, they make lines.



- **Composite numbers** have more than 2 factors. They make arrays other than in one line.
- **Even numbers** can all make arrays with 2 on one side (as one factor). **Odd numbers** can't.
- **Square numbers** can make square arrays.



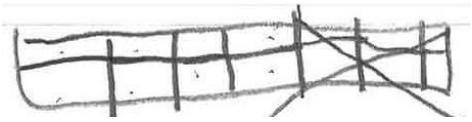
Structural stages

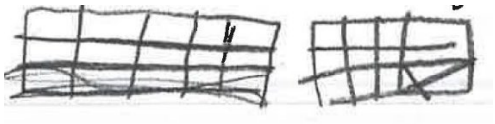
Here are some drawings to show what to look out for if you are worried about your child. We want children to be in the "structural" stage at this point, rather than one of the earlier phases. Please contact your child's teacher if you are concerned.

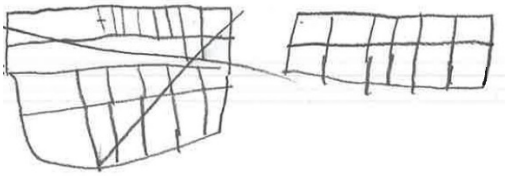
Look at your child's drawings to determine what structural level they are at. Emphasise moving to the next structural stage rather than drawing larger amounts. For teachers: Joanne Mulligan has more information on developing structural thinking in the PASMAP research available online.

Each of the drawings below is of a **tens frame** (rectangle with 2 rows of 5), drawn by a child who is familiar with tens frames but can't see one. Each drawing was completed by a child aged between 5 and 8.

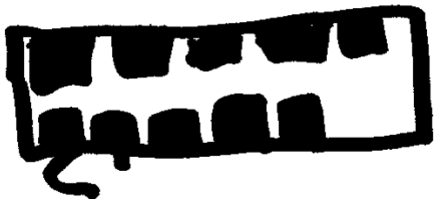
Emergent: correct number of rows or columns, but not both, or just 10 in one line but without 2 rows



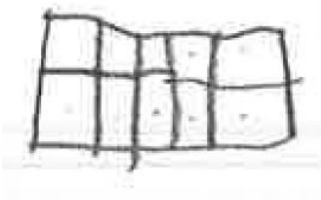


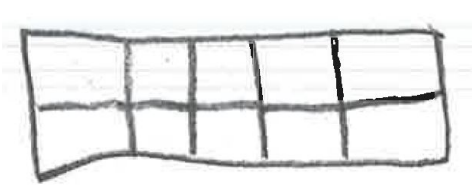


Partial structural: can draw 10, but not also keep the structure of rows and columns, often this means 2 rows of 5 but not having the squares touching



Structural: both drawings show structural thinking, however the dots on the images show that the child needed to check that there really were 10





Teacher Overview

Students will be thinking about arrays to develop multiplicative thinking. They will also be focusing heavily on the distributive property and multiplying single-digit by two-digit numbers. Time with families is well-spent in becoming fluent in multiplication facts.

Students need experience in creating and drawing objects arranged into structures in order to develop strong mental objects for numbers. The term "mental objects" refers to being able to move things around like real objects in your mind. You have to be able to manipulate them, not just picture them. This will help students later with place value, algebraic thinking, multiplicative thinking, understanding fractions and lots more.

- Construction and deconstruction of models provides experiences that help students to build perceptive understanding of multiplicative relationships.
- While both arrays and groups can be used to represent multiplication and division, arrays are much more powerful. They show both factors and the multiple at the same time. They connect "counting in" (e.g. number in each row), with multiplying. They can be used later to show properties of numbers such as "square", "prime", "composite" and to also demonstrate the commutative and distributive laws. They are incredibly important for algebra and are also linked with an understanding of fractions and probability. **Please don't skip arrays**.
- We need to move on from simply drawing arrays made of lots of boxes to drawing rectangles and cutting them into pieces. This helps develop the structural thinking needed for division.
- Using the distributive law allows us to multiply larger numbers, and also leads into algebraic multiplication in the next year or two.

What to emphasise

If you have time online with a webcam
Work on having students draw rectangles and cut them to make arrays rather than just drawing everything individually. Check that the parents understand how the number games for the week work and make sure that you ask the student if they have played them yet.

If you have only email or phone contact
Check that parents have read the "What you need to know this week" section. Check that they understand the importance of using the number tasks and interleaving sheet so that students retain what they have learned and think regularly about number.

Tracking student achievement

- Can the student solve problems using all four operations (NB at least 2-digit by 2-digit multiplication)? Tick **N3C.** This is Lesson 2.
- Can the student make connections between powers of 10 and multiplication with decimals? Tick **N6C**. This is Lesson 3.
- By year 4 students were meant to be able to recall all single-digit multiplication facts. As many students need more practice, we have included multiplication grids as well.

Monday: At-Home Investigation

You will need:

- Grid paper that is provided (you will need multiple copies)
- Coloured pencils

Steps:

1. Make sure you have read "What you need to know this week" so that you know what to emphasise with your child.
2. Read the sheet to your child. Encourage your child to come up with many different rectangles or arrays using 60 squares (e.g. 6x10, 15x4)
3. Discuss what your child found out from the arrays. Keep in mind the ideas from the "What you need to know this week" section so that you can ask questions that are appropriate to the issues identified. Try to encourage your child to explain how they have solved the problem.
4. 100 is another number with lots of factors. That is because it also has 4 prime factors (2, 2, 5, 5). Others include 54 (3, 3, 3, 2), 64 (2, 2, 2, 2, 2, 2), 96 (2, 2, 2, 2, 2, 3) and 90 (3, 3, 2, 5). To find them, simply try multiplying lots of small prime numbers together, then seeing what combinations you can come up with. Having different combinations of numbers gives more interesting factors.

Extension:

What if you had to build rectangular prisms with 60 blocks rather than just drawing flat rectangles? How many unique 3D shapes could you build (if you turn the shape sideways then it still counts as the same shape)?

Please note:

By the end of year 4, your child was expected to recall all single digit multiplication facts (up to 10x10). Now is a great time to practise these with your child so that their previous knowledge is not lost, or to build greater recall if they have not yet mastered basic facts.

At-Home Investigation

How many different arrays can we make with 60 squares or blocks?

Draw the arrays and label them

Use the grid paper to draw as many unique arrays using 60 squares as you can. In this activity, 6x10 is considered to be the same as 10x6, so you only need to draw it once. You will probably need multiple sheets of grid paper and will also need to cut it and stick it together to make the right sizes.

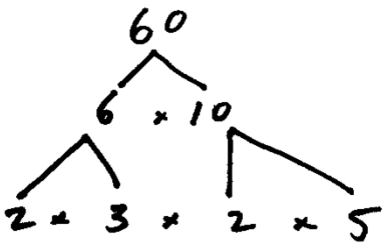
For each array, label the factors (the sides). Write each set of factors here:

Think it through

Can you find 2 other numbers between 50 and 100 that have as many or more factors than 60? Write the numbers and sketch the arrays and list the factors here:

Apply your thinking:

The factors of 60 can be broken down further into prime factors using a factor tree. The more prime factors a number has, the more factors it will have in general. Look at the factor tree below for 60. Use the same thinking to make a prime factor tree for the numbers you looked at for the previous question.



Teacher Overview

This is a ***Problem Solving and Reasoning*** task.

The emphasis is on *modelling* arrays and exploring how prime factors relate to factor combinations. We want students to think about *similarities*, *differences* and *patterns* or *characteristics*. There is also an emphasis on *generalising* – such as realising that the orientation of an array does not change the total – and exploring the distributive property to solve problems.

Comparing arrays of 60 is something that you can do via a webcam. Children can also hold up their drawings of arrays if they do them in pen. The extension task, explained on the sheet for families, is also great to play with via a webcam. It involves making 3D structures from 60 cubes. Isometric dot paper has been included online so that you can download it to use for extension as appropriate.

Watch out for:

- Adults drawing for the children
- Not understanding what prime means
- Not understanding what factors are
- Double-ups
- No structure to the factors: no way of organising the information to determine whether they have found them all.

Good questions to prompt thinking:

- Show me your arrays
- How have you organised your arrays so that you know you have found all the combinations?
- How about organising your list from the smallest factor through to the largest so that we can find any that we may have missed?

Students requiring support:

- Reduce the number of squares to 36
- Use the Year 3 or 4 problem focusing on Lego and working out the dots instead
- Find one set of factors that they know, then try making one of the numbers smaller or larger until you find another one that works
- Use a calculator to try combinations

Extension:

- How can we use combinations of the prime factors to make sure that we have all the possible combinations for 60?

Tuesday: Connecting Lesson

Number task for 10-15 minutes: *Finding multiples*

Roll a dice 4 times to get 4 numbers, or just pick any 4 numbers between 1 and 8.

1. List all the different multiples can you make using any 2 of the 4 numbers.
2. What would you get if you multiply all 4 numbers?
3. Which of the numbers are prime, which are composite?

Alternative: complete one of the multiplication grids on the following page.

Worksheet task: 20-30 minutes
This lesson provides an opportunity for your child to extend their knowledge of multiplication by tens and ones to include multiplying 2-digit numbers together. There is a diagram showing the array which should be helpful. This diagram makes use of the distributive property which is explained in the introduction. Please give your child just the first sheet rather than providing all three at once. Use the second sheet only once they have had a try thinking about a model. The third sheet provides an explanation and some practice questions.

If this lesson proves too difficult for your child, try revising first by using the lesson from the Year 5 work program on multiplying by tens and ones first (Wednesday's lesson for this week).

Please note: being able to multiply two-digit numbers by two-digit numbers is a requirement for the "C" standard for year 6.

Multiplication practice grids:

	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									

	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									

	2	3	4	5	6	7	8	9	10
2									
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9									
10									

	2	3	4	5	6	7	8	9	10
2									
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7									
8									
9									
10									

Multiplying two-digit numbers

Use your previous knowledge of multiplication to help you to solve the following problems.

What we already know how to do:

$$\begin{array}{rr} & 34 \\ \times & 3 \text{ ones} \\ \hline & \\ \hline \end{array}$$

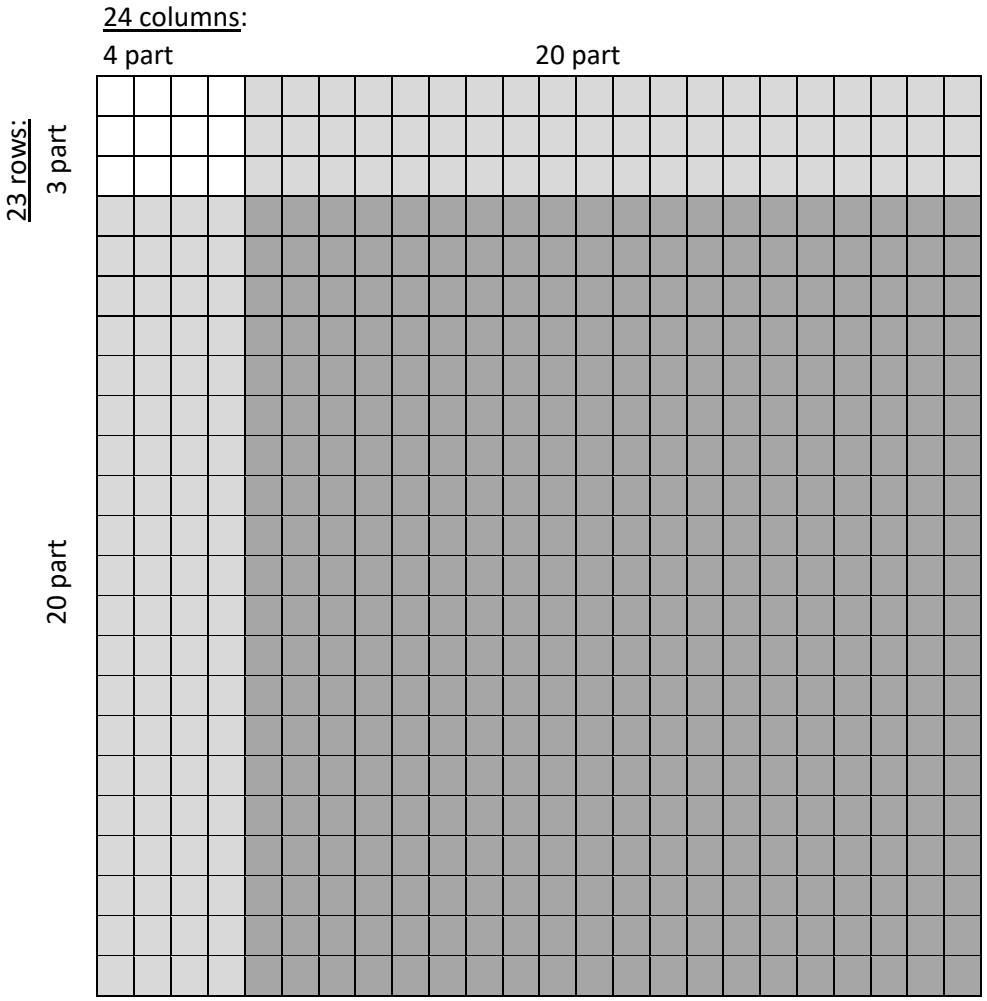
Sketch what this looks like, and break your array into the tens and ones parts. You do not need to show every single square, just make the rectangle into roughly the right proportions.

What about if we were multiplying by tens instead of ones?

$$\begin{array}{rr} & 34 \\ \times & 3 \text{ tens} \\ \hline & \text{tens} \\ \hline \end{array} \qquad \begin{array}{rr} & 34 \\ \times & 30 \text{ ones} \\ \hline & \text{ones} \\ \hline \end{array}$$

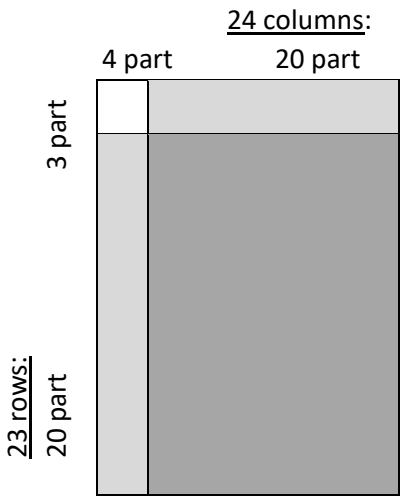
What would it look like to multiply 34 by three tens and also by three ones (34 x 33)?
Use the space below to sketch this very simply and work it out:

Here is an array for 24 x 23. Use the parts to see if you can work out what the answer is.



Work out how many squares there are in each different part of the diagram above.
Do you need to count every square or is there an easy way to work out how many there are?
Use the space below to show your calculations for each part of the array, then work out how many squares there are altogether.

Using a simplified array or written strategies is faster than drawing all of the boxes in an array. Look at the simplified array below and the written strategy. See if you can work out how they are related. Draw lines between the parts in the written strategy that relate to the array.



Written Strategy:

$$\begin{array}{r} 24 \\ \times\ 23 \\ \hline 12 \\ 60 \\ 80 \\ 400 \\ \hline \mathbf{552} \end{array}$$

Explain how the written strategy works using your own words:

Try solving the questions below. Check that you are right by sketching a simplified array.

$$\begin{array}{r} 35 \\ \times\ 23 \\ \hline \\ \\ \\ \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 28 \\ \times\ 35 \\ \hline \\ \\ \\ \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 34 \\ \times\ 21 \\ \hline \\ \\ \\ \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 25 \\ \times\ 43 \\ \hline \\ \\ \\ \\ \hline \\ \hline \end{array}$$

Teacher Overview

This is a ***Problem Solving*** lesson. It is designed to help students *develop* a visual *model* for multiplication BEFORE showing them the array. If possible, please try to guide parents to provide only page 1 at the start as page 2 provides the explanation. If that doesn't work, the child will still need to use *deductive* processes to determine how the model works which is better than just telling them.

Please note, the Australian Curriculum only requires students to use written processes. It does not specify which processes to use. An array model is efficient and accurate, and it better links to solving quadratic equations in years 9/10 than the standard two-line algorithm. It also leads into multiplication of fractions and decimals. While you are welcome to also teach the two-line algorithm, this method has some definite advantages for the longer term.

Wednesday: Connecting Lesson

This lesson asks your child to connect what they have learned over the past two days to work out the pattern for multiplying with decimal numbers. Feel free to provide a calculator for your child (including a calculator app on a phone) to take the pain out of the calculation side and focus instead on the patterns.

Number game for 10-15 minutes: *Array fun*
You will need: a print out of the grid from Monday, 2 colours of pencil, one or two dice.

1. Player one rolls the two dice (or one dice two times). The numbers rolled are the length and width of your array to colour! (e.g. a 4 and a 3 would need a 4 x 3 array) You can turn it sideways to fit. Colour your array on the grid, then it is the other player's turn.
2. The player who wins is the last player who can draw their array.

Alternative: complete another multiplication grid

Worksheet task: 20 minutes
This lesson is following on from what your child learned yesterday about multiplying by tens and ones. The purpose of the lesson is to **connect** what they have learned about patterns in multiplication with patterns for multiplying by decimal numbers.

Tricky part: The difficulty that most children seem to have with decimal numbers is that they want to line up the numbers and decimal points and put decimal points in each line of working. This is irrelevant and confusing. Please instruct your child to ignore the decimal points at the start and just multiply the numbers as if there were no decimal places at all. At the end, work out the number of decimal places needed in the answer. Use the patterns provided to figure out how.

Here are some of the patterns to help:

- 3 x 5 = 15 (no decimal places)
- 0.3 x 5 = 1.5 (1 decimal in question = 1 in the answer)
- 3 x 0.5 = 1.5 (1 decimal in question = 1 in the answer)
- 0.3 x 0.5 = 0.15 (2 decimals in question = 2 in the answer)
- 0.03 x 5 = 0.15 (2 decimals in question = 2 in the answer)
- 0.00003 x 0.005 = 0.00000015 (8 decimals in question = 8 in the answer)

Please note: being able to multiply numbers with decimal places is a requirement for the "C" standard for year 6.

D12. Decimals in multiplying

TR p92

Multiplying with decimal numbers is very similar to multiplying with whole numbers. Examine the following examples and see if you can find the pattern between the decimals in the terms and the decimals in the answer.

Example 1:

12.4	$1^{1}9.3$	$2^{1}6.3$	$2^{1}4.2$
x 2	x 2	x 3	x 4
24.8	38.6	78.9	96.8

What is the pattern?

Apply this pattern to answer the questions below.

16.3	32.5	17.6	15.3
x 2	x 2	x 4	x 7

Example 2:

1.24
x 2
2.48

What is the pattern?

Apply this pattern to answer the following questions:

1.63	2.73	1.35	2.87
x 2	x 2	x 4	x 5

Make up a rule to describe how to know where to put the decimal points when multiplying:

Backwards Question:

Put the decimal points into the following equation. What other possibilities are there? Write as many as you can:

1 2 4 x 2 = 0. 2 4 8

D13. Decimals in multiplying 2

TR p93

Multiplying with decimal numbers is very similar to multiplying with whole numbers. Examine the following examples and see if you can find the pattern between the decimals in the terms and the decimals in the answer.

Examples:

5.4	0.54	54	54
x 2.3	x 2.3	x 2.3	x 0.23
162	162	162	162
1080	1080	1080	1080
12.42	1.242	124.2	12.42

What is the pattern?

Apply this pattern to answer the questions below.

56	57	5.6	7.4
x 0.43	x 5.8	x 0.82	x 6.3

Check your answers with a calculator. If you are still having difficulty seeing the pattern, go back and look at the examples again. Compare the total number of decimal places in the question with the total number of decimal places in the answer. Show your answers to your teacher before continuing.

Make up a rule to describe how to know where to put the decimal points when multiplying:

Backwards Question:

Put the decimal points into the following equation. What other possibilities are there? Write as many as you can:

54 x 23 = 1.242

Teacher Overview

This is a ***Reasoning and Understanding*** lesson. It is designed to determine if students can adapt and transfer the strategies and processes they have learned for multiplication to calculate multiplication of decimal numbers. In this lesson, students look for similarities and differences, come up with a theory, test out their ideas and generalise their findings.

If you have time online with students, focus on the connection between the number of decimal places in the question and the number of places in the answer. Watch out for students wanting to line up the decimal places or put a decimal in each line of working.

Other considerations:

- Check that the student has completed the number tasks or multiplication grids and remind parents that it is important if they haven't used them.
- Ask the parent to work on the recall of multiplication facts grid at least once each week. If the child is ok with it, have them time themselves, then plot the time over each week that school is closed. Hopefully they will improve significantly. Do not make this a requirement though as some children respond with heightened anxiety.

Important note going forward:
Once this lesson has been completed, you can adapt the multiplication grids provided to include decimal numbers. This has the advantage of still building fluency in basic facts but also reinforcing multiplication with decimal places.

Thursday: Interleaved Practice Questions

Why we are using mixed up questions:
In this lesson your child will be reviewing a range of skills that they have learned previously. Each question is unrelated to the previous question, because we want your child to have to *think hard* about what to do. Mixing up questions like this, rather than just practising related questions, has been shown in research to improve student retention of concepts by 60% over a 4 month period.

What to expect:
Your child will probably have forgotten how to complete quite a few of the questions. If needed, change the numbers in each question to make them easier because this will still require your child to think hard and remember a process. If they still can't work it out, feel free to show them, but try using different numbers rather than the exact same question. There are answers to each question on the website in case you get stuck.

Interleaved practice

Number:

1. Complete the following number sequence and describe the rule used:

   8.73, 8.83, _____, _____, _____, 9.23, _____,

2. 52 342 - _____ = 30 527

3. Write the numbers 20 to 35 in the correct box.

Prime Numbers	Composite Numbers

4. Read this number and say it: 4 051 738. Round it to the nearest 10, the nearest 100, the nearest 1000, and the nearest 10 000.

5. What is $^{3}/_{5}$ of 30? Work it out in two different ways.

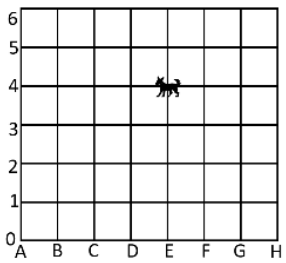
Measurement/Geometry:

6. Use a measuring jug from your kitchen. Find one container that holds less than your measuring jug and one that holds more than it. Use the measuring jug to find out how much water, each container will hold. Record your findings in millilitres and then in litres.

7. You need to catch a bus to school from Main Road and be there in time for sports practice at 8:30am. Use the timetable to decide when you need to be at your bus stop and why.

Stops	Central line	Central line	Central line	Central line
Junction Street	7:46am	8:01am	8:16am	8:31am
Main Road	7:48am	8:03am	8:18am	8:33am
Anzac Avenue	7:52am	8:07am	8:22am	8:37am
Central School	7:55am	8:10am	8:25am	8:40am
Central Shopping Centre	8:01am	8:16	8:31am	8:46am

8. Write the grid reference for the dog. Draw another dog at C,2 on the grid.



Chance/Data:

8. In this graph, each block represents 5 people.
   How many people like each colour?
   What else can you tell from the information in the graph? Write 2 **true** statements.

Teacher Overview

The questions on this worksheet are drawn from the “C standard” of the Achievement Standard. See your tracking sheet for more detail. Each week the interleaved questions will get a little harder, and more concepts will be reviewed throughout the program as we teach that concept. We have included answers to these questions on the B2FMaths@Home so that parents can find them if needed.

Support for struggling students:
You might like to try the Interleaved questions from a lower year level, or simply reduce the numbers in the questions. You might also give the student the answer then ask them to work out how the answer was obtained.

Friday: Connecting Lesson

Distributive Law

In the previous lessons this week we have considered multiplying by tens and also multiplying by ones. In this lesson your child will put these ideas together unpack a very important property of multiplication called the distributive law or the distributive property. Given the work that they have done on multiplying by tens and ones on Wednesday, this worksheet should be relatively simple.

Please consider reviewing the Lego poster from earlier this week and checking that your child is remembering those particular number facts more easily.

Here are a few quick grids of the facts that students tend to find the most difficult to recall. Feel free to use them for practice instead. They have fewer calculations, but are the harder ones.

	3	4	6	7	8
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4					
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D8. Distributive Law

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Often using mental strategies when multiplying is quicker than using a calculator. Look at the following examples, and work out how the 'distributive law' is being applied.

Distributive Law: Used when breaking up an equation into smaller parts makes it easier.

Examples:

- 19 x 5 = (9 x 5) + (10 x 5) = 45 + 50 = 95
- 36 x 9 = (30 x 9) + (6 x 9) = 270 + 54 = 324

Try these:

33 x 4 = ( ) + ( ) = _____ + _____ = _____

23 x 5 = ( ) + ( ) = _____ + _____ = _____

35 x 7 = ( ) + ( ) = _____ + _____ = _____

54 x 6 = ( ) + ( ) = _____ + _____ = _____

37 x 2 = ( ) + ( ) = _____ + _____ = _____

What do you think the distributive law does?

How do you know?

Where could you use this?

Backwards Questions:

Try to use the distributive law together with what you have learned about extending multiplication facts to solve the following:

330 x 4 = ( ) + ( ) = _____ + _____ = _____

23 x 50 = ( ) + ( ) = _____ + _____ = _____

350 x 70 = ( ) + ( ) = _____ + _____ = _____

Look up the Distributive Law using a mathematical dictionary and write a definition for it using your own words:

Teacher Overview

This is a ***Reasoning*** lesson. It is designed to determine if students can apply the strategies and processes they have learned for multiplication to calculate multiplication when one of the numbers has two digits. In this lesson, the students work through a two-step process to multiply rather than a single line process. This helps develop the thinking necessary for later algebraic processes, so it is necessary to teach even if it is not as efficient as the single line process.

If you have time online with students, focus on the connection between multiplying by tens and ones. Try other numbers as well. Draw the grids if needed.