Work Program for B2FMaths@Home

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## How to use this work program

## Accessing the online resources

To access the online resources, please go to: https://www.backtofrontmaths.com.au/b2fmathshome

## Running the program each week

Each week is designed with five maths lessons so that you can do it each day. Different days have different types of lessons to make sure that students experience the kind of thinking that they need to continue growing in maths. The types of lessons include:

- At-home investigation: This is a hands-on task where students explore a new idea before they are taught that skill. They need to come up with an idea to try to solve the problem, try out their idea, decide if it worked or not, try again if needed, and explain what they did. If your child has time with your teacher with a webcam, the teacher will generally be doing this lesson with your child. This is the lesson that will require the heaviest input from you to help your child think through an idea and generally requires the use of some hands-on materials that are listed in the information page.
- Connecting lesson: This type of lesson has questions that lead students to develop their ideas and learn a new skill. It should be fairly easy for a student to do, but you will need to be available to read the question to your child as needed, encourage them to think further, and make sure that they complete the work. Most of these lessons will include 10 minutes of practising number operations or concepts through activities or games.
- Interleaved practise lesson: This type of lesson provides 8-10 questions from different areas of maths so that students practise remembering what they have previously been taught. Some of the questions may not be easy for your child, so feel free to help whenever you see them struggling.
- Generalising lesson: This lesson contains some extension material for use if your child found the week's lessons too easy. If you would prefer, you can spend this lesson playing more of the number games that are included in the connecting lesson or giving your child time to complete any of the lessons that they have not yet done.


## Getting help

The website above will have answers to frequently asked questions as well as videos to help you successfully teach your child at home. If you have further questions or need support, please contact your child's teacher directly using the contact details that they have provided to you. If they can't answer your questions, they will contact the B2FMaths@Home team directly to get an answer within 3 days.

## What you need to know this week

## Week overview

This week we are teaching the concept arrays and counting patterns. We will particularly be focusing on arrays of objects arranged into a grid-like pattern (e.g. tiles or the top of Lego blocks). This model for multiplication and division has links with many other concepts in later years, such as area, volume, fractions and helps develop a firm foundation for understanding algebra.

## Students need to understand:

- Distributive Property: Arrays can be easily split to make multiplying large numbers easier.



## 4 sevens is the same as

## 4 fives and 4 twos

$$
4 \times 7=4 \times 5+4 \times 2
$$

- Commutative Property: Arrays can be rotated to demonstrate that $4 \times 3=3 \times 4$.



## 4 threes $=3$ fours

- Factors are the length and the width of an array. The multiple is the amount altogether.
- Prime numbers, like 7, can only be made by multiplying 1 by itself. That means that they only have 2 factors: themselves and 1 . When we arrange prime numbers into arrays, they make lines.

- Composite numbers have more than 2 factors. They make arrays other than in one line.
- Even numbers can all make arrays with 2 on one side (as one factor). Odd numbers can't.
- Square numbers can make square arrays.



## Structural stages

Here are some drawings to show what to look out for if you are worried about your child. We want children to be in the "structural" stage at this point, rather than one of the earlier phases. Please contact your child's teacher if you are concerned.

Look at your child's drawings to determine what structural level they are at. Emphasise moving to the next structural stage rather than drawing larger amounts. For teachers: Joanne Mulligan has more information on developing structural thinking in the PASMAP research available online.

Each of the drawings below is of a tens frame (rectangle with 2 rows of 5 ), drawn by a child who is familiar with tens frames but can't see one. Each drawing was completed by a child aged between 5 and 8 .

Emergent: correct number of rows or columns, but not both, or just 10 in one line but without 2 rows


Partial structural: can draw 10, but not also keep the structure of rows and columns, often this means 2 rows of 5 but not having the squares touching


Structural: both drawings show structural thinking, however the dots on the images show that the child needed to check that there really were 10


## Teacher Overview

Students will be thinking about arrays to develop multiplicative thinking. They will also be focusing heavily on the distributive property and multiplying single-digit by two-digit numbers. Time with families is well-spent in becoming fluent in multiplication facts.

Students need experience in creating and drawing objects arranged into structures in order to develop strong mental objects for numbers. The term "mental objects" refers to being able to move things around like real objects in your mind. You have to be able to manipulate them, not just picture them. This will help students later with place value, algebraic thinking, multiplicative thinking, understanding fractions and lots more.

- Construction and deconstruction of models provides experiences that help students to build perceptive understanding of multiplicative relationships.
- While both arrays and groups can be used to represent multiplication and division, arrays are much more powerful. They show both factors and the multiple at the same time. They connect "counting in" (e.g. number in each row), with multiplying. They can be used later to show properties of numbers such as "square", "prime", "composite" and to also demonstrate the commutative and distributive laws. They are incredibly important for algebra and are also linked with an understanding of fractions and probability. Please don't skip arrays.
- We need to move on from simply drawing arrays made of lots of boxes to drawing rectangles and cutting them into pieces. This helps develop the structural thinking needed for division.
- Using the distributive law allows us to multiply larger numbers, and also leads into algebraic multiplication in the next year or two.


## What to emphasise

## If you have time online with a webcam

Work on having students draw rectangles and cut them to make arrays rather than just drawing everything individually. Check that the parents understand how the number games for the week work and make sure that you ask the student if they have played them yet.

## If you have only email or phone contact

Check that parents have read the "What you need to know this week" section. Check that they understand the importance of using the number tasks and interleaving sheet so that students retain what they have learned and think regularly about number.

## Tracking student achievement

- Can the student solve problems including multiplying single digit numbers by double digit numbers? Tick N1C. Applying this to $2 \times 2$ digits is a B .
- Can the student identify and describe factors and multiples? Tick N3C.
- By year 4 students were meant to be able to recall all single digit multiplication facts. As many students need more practice, we have included multiplication grids as well.


## Monday: At-Home Investigation

## You will need:

- Grid paper that is provided (you will need multiple copies)
- Coloured pencils


## Steps:

1. Make sure you have read "What you need to know this week" so that you know what to emphasise with your child.
2. Read the sheet to your child. Encourage your child to come up with many different rectangles or arrays using 60 squares (e.g. 6x10, 15x4)
3. Discuss what your child found out with them. Keep in mind the ideas from the "What you need to know this week" section so that you can ask questions that are appropriate to the issues identified. Try to encourage your child to explain how they have solved the problem.
4. 100 is another number with lots of factors. That is because it also has 4 prime factors $(2,2,5,5)$. Others include $54(3,3,3,2), 64(2,2,2,2,2,2), 96(2,2,2,2,2,3)$ and $90(3,3,2,5)$. To find them, simply try multiplying lots of small prime numbers together, then seeing what combinations you can come up with. Having different combinations of numbers gives more interesting factors.

## Extension:

What if you had to build rectangular prisms with 60 blocks rather than just drawing flat rectangles? How many unique 3D shapes could you build (if you turn the shape side ways then it still counts as the same shape)?

## Please note:

By the end of year 4, your child was expected to recall all single digit multiplication facts (up to $10 \times 10$ ). Now is a great time to practise these with your child so that their previous knowledge is not lost, or to build greater recall if they have not yet mastered basic facts.

## At-Home Investigation

How many different arrays can we make with 60 squares or blocks?

## Draw the arrays and label them

Use the grid paper to draw as many unique arrays using 60 squares as you can. In this activity, $6 \times 10$ is considered to be the same as $10 \times 6$, so you only need to draw it once. You will probably need multiple sheets of grid paper and will also need to cut it and stick it together to make the right sizes.

For each array, label the factors (the sides). Write each set of factors here:

## Think it through

Can you find 2 other numbers between 50 and 100 that have as many or more factors than 60 ?
Write the numbers and sketch the arrays and list the factors here:

## Apply your thinking:

The factors of 60 can be broken down further into prime factors using a factor tree. The more prime factors a number has, the more factors it will have in general. Look at the factor tree below for 60 . Use the same thinking to make a prime factor tree for the numbers you looked at for the previous question.



## Teacher Overview

## This is a Problem Solving and Reasoning task.

The emphasis is modelling arrays and exploring how prime factors relate to factor combinations. We want students to think about similarities, differences and patterns or characteristics. There is also an emphasis on generalising - such as realising that the orientation of an array does not change the total - and exploring the distributive property to solve problems.

Comparing arrays of 60 is something that you can do via a webcam. Children can also hold up their drawings of arrays if they do them in pen. The extension task, explained on the sheet for families, is also great to play with via a webcam. It involves making 3D structures from 60 cubes. Isometric dot paper has been included online so that you can download it to use for extension as appropriate.

## Watch out for:

- Adults drawing for the children
- Not understanding what prime means
- Not understanding what factors are
- Double-ups
- No structure to the factors: no way of organising the information to determine whether they have found them all.


## Good questions to prompt thinking:

- Show me your arrays
- How have you organised your arrays so that you know you have found all the combinations?
- How about organising your list from the smallest factor through to the largest so that we can find any that we may have missed?


## Students requiring support:

- Reduce the number of squares to 36
- Use the Year 3 or 4 problem focusing on Lego and working out the dots instead
- Find one set of factors that they know, then try making one of the numbers smaller or larger until you find another one that works
- Use a calculator to try combinations


## Extension:

- How can we use combinations of the prime factors to make sure that we have all the possible combinations to 60 ?


## Tuesday: Connecting Lesson

Number task for 10-15 minutes: Finding a total
This task is the same as last week

Choose a composite number between 10 and 50.

Roll a dice 4 times to get 4 numbers, or just pick any 4 numbers between 1 and 8 .

Try to use those numbers to get as close as possible to your target number.

## Rules:

- Not all 4 numbers have to be used
- A number can only be used once
- Use any operation you like ( $+-\mathrm{x} \div$ ) and any others that you know (e.g. powers or square roots, ! etc.)

Try at least 3 numbers.

## Worksheet task: 10 minutes

This lesson provides an opportunity for your child to review multiplying by 10 and 100 before we move onto multiplying multi-digit numbers next lesson. It should be quick. Please also complete one of the multiplication grids on the following page to identify any multiplication facts that your child finds difficult to recall, then complete the Lego or array poster below.

## Lego arrays poster task: 15 minutes

Use Lego bricks to represent any multiplication facts that your child finds hard to remember, or draw them on the grid paper provided. Take a photo of what you have made. Print it out and write numbers onto the sides as appropriate. Display your photo somewhere prominent to help build recall.


## Multiplying by 10 and 100

You have previously found some patterns for multiplying by ten. In this activity you will extend these patterns to multiply very large numbers.

Work out the following questions, then use a calculator to check afterwards:

| Basic fact: | Extension of fact: | Further extension: | Check with the <br> calculator: |
| :--- | :--- | :--- | :--- |
| Example1: <br> $6 \times 7=42$ | 6 tens $\times 7=42$ tens | $60 \times 7=420$ |  |
| Example $2:$ <br> $4 \times 8=32$ | $4 \times 8$ hundreds $=32$ hundreds | $4 \times 800=3200$ |  |
| $3 \times 7=$ | 3 tens $\times 7=$ |  |  |
| $9 \times 3=$ | $9 \times 3$ tens $=$ |  |  |
| $2 \times 6=$ | $2 \times 6$ hundreds $=$ |  |  |
| $5 \times 8=$ | 5 tens $\times 8$ tens $=$ |  |  |

What is the pattern? How many places have the original numbers moved away from the ones?

Use it to complete the table below:

| Basic fact: | Extension of fact: | Further Extension: | What is the pattern? |
| :--- | :--- | :--- | :--- |
| $3 \times 8=$ | 3 tens $\times 8=$ |  |  |
|  | 9 hundreds $\times 7=$ | $2 \times 40=$ |  |
|  | $3 \times 5$ hundreds $=$ | $60 \times 40=$ |  |
|  | 9 tens $\times 6$ hundreds $=$ |  |  |
|  |  |  |  |

## Extension:

What would you multiply 90 by to get 630?

Multiplication practice grids:

|  | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
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## Teacher Overview

This is a Reasoning lesson. It is designed to see if students can apply the strategies they have learned for multiplication of single-digit facts to multiplying by 10 and 100. Hopefully this lesson is mostly a review of concepts, but it will help families to understand which facts their child needs to focus on learning.

To help students retain the information, make sure that they have explained their strategies to their parents. If you have time online with students, emphasise connections between multiplication facts such as the commutative property and how to calculate efficiently using the distributive property.

The Lego task is an optional extra but is great for helping families to see the connection between "counting in" and the array structure. It also useful for building confidence in adults and enjoyment for children.

## Wednesday: Application Lesson

This lesson allows your child to apply what they have learned over the past two days to work out how to multiply by numbers with multiple digits.

## Number game for 10-15 minutes: Array fun

You will need: a print out of the grid from Monday, 2 colours of pencil, one or two dice.

1. Player one rolls the two dice (or one dice two times). The numbers rolled are the length and width of your array to colour! (e.g. a 4 and a 3 would need a $4 \times 3$ array) You can turn it sideways to fit. Colour your array on the grid, then it is the other player's turn.
2. The player who wins is the last player who can draw their array.

Alternative: complete another multiplication grid

## Worksheet task: $\mathbf{2 0}$ minutes

This lesson is following on from what your child learned yesterday about arrays. The purpose of the lesson is to connect what they have learned about the arrays for single digit numbers and for multiplying by ten, with a number that has both tens and ones. This requires the Distributive Property.

Please note, in this activity the children will be recording the multiplication by tens and ones on separate lines then adding them together. We understand that it is more efficient to put these together on a single line, however teaching multiplication like this leads to greater understanding of the Distributive property which is needed for algebra from year 7. By slowing down the process rather than teaching the more efficient strategy at the start, we are building a foundation for understanding that will help prepare the children for better understanding in high school.

## Multiplying by tens and ones

Multiplying by tens and ones is easy once we can think in arrays. In this lesson we will learn how to break two-digit numbers into tens and ones to make them easier to multiply.

## Use grid paper to draw $7 \times 35$

1. The 35 part can be separated into tens and ones. Draw a line to separate the 35 into 30 and 5 .
2. Find the part that is $7 \times 30$. How many squares are there?
3. How is this similar to $7 \times 3$ ?
4. Find the part that is $7 \times 5$. How many squares are there?
5. So how many squares are there altogether?

## Use grid paper to draw $9 \times 24$

1. The 24 part can be separated into tens and ones. Draw a line to separate the tens and ones.
2. Find the part that is $9 \times 20$. How many squares are there here?
3. How is this similar to $9 \times 2$ ?
4. Find the part that is $9 \times 4$. How many squares are there here?
5. So how many squares are there altogether?

The equation below represents the first question that you worked out ( $7 \times 35$ ). Look at it and try to find the $7 \times 5$ part and the $7 \times 30$ part.


Try these:

|  | 5 | 6 |  |  | 7 |  | 8 | 3 |  | 4 | 9 |  | 6 | 8 |  | 9 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| X |  | 5 | X |  | 6 | X |  | 4 | X |  | 2 | x |  | 7 | x |  |  |

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## Teacher Overview

This is a Reasoning lesson. It is designed to determine if students can apply the strategies and processes they have learned for multiplication to calculate multiplication when one of the numbers has two digits. In this lesson, the students work through a two-step process to multiply rather than a single line process. This helps develop the thinking necessary for later algebraic processes, so it is necessary to teach even if it is not as efficient as the single line process.

If you have time online with students, focus on the connection between multiplying by tens and ones. Try other numbers as well. Draw the grids if needed.

## Other considerations:

- Check that the student has played the number games or multiplication grids and remind parents that it is important if they haven't played them with their child.
- Ask the parent to work on the recall of multiplication facts grid at least once each week. If the child is ok with it, have them time themselves, then plot the time over each week that school is closed. Hopefully they will improve significantly. Do not make this a requirement though as some children respond with heightened anxiety.


## Thursday: Interleaved Practice Questions

## Why we are using mixed up questions:

In this lesson your child will be reviewing a range of skills that they have learned previously. Each question is unrelated to the previous question, because we want your child to have to think hard about what to do. Mixing up questions like this, rather than just practising related questions, has been shown in research to improve student retention of concepts by $60 \%$ over a 4 month period.

## What to expect:

Your child will probably have forgotten how to complete quite a few of the questions. If needed, change the numbers in each question to make them easier because this will still require your child to think hard and remember a process. If they still can't work it out, feel free to show them, but try using different numbers rather than the exact same question. There are answers to each question on the website in case you get stuck.

## Interleaved practice

Number:

1. Complete the following number sequence:
8.73, 8.83, $\qquad$ , $\qquad$
$\qquad$ 9.23, $\qquad$
2. 2342 - $\qquad$ = 1127
3. What number is 1 more than 495099 ?

Now write the number that is 10 more and the number that is 100 more and the number that is 1000 more and the number that is 10000 more than 495099.
4. Read this number and say it: 4051 738. Round it to the nearest 10, the nearest 100 , the nearest 1000, and the nearest 10000.
5. Share 30 counters to show halves. Draw the halves on the other side of this sheet.

What other fractions can you make? How will you know if you have found them all?

## Measurement/Geometry:

6. Use a measuring jug from your kitchen. Find one container that holds less than your measuring jug and one that holds more than it. Use the measuring jug to find out how much water, each container will hold. Record your findings.
7. If you begin your lunch break at 13:20 and take 45 minutes to make and eat it, what time will you finish it? Write your answer in 24 -hour time and analogue time? (reading a clockface)
8. Write the grid reference for the dog. Draw another dog at $C, 2$ on the grid.


Chance/Data:
9. In this graph, each block represents 5 people.

How many people like each colour?
What else can you tell from the information in the graph? Write 2 true statements.


## Teacher Overview

The questions on this worksheet are drawn from the "C standard" of the Achievement Standard. See your tracking sheet for more detail. Each week the interleaved questions will get a little harder, and more concepts will be reviewed throughout the program as we teach that concept. We have included answers to these questions on the B2FMaths@Home so that parents can find them if needed.

## Support for struggling students:

You might like to try the Interleaved questions from a lower year level, or simply reduce the numbers in the questions. You might also give the student the answer then ask them to work out how the answer was obtained.

## Friday: Connecting Lesson

## Distributive Law

In the previous lessons this week we have considered multiplying by tens and also multiplying by ones. In this lesson your child will put these ideas together unpack a very important property of multiplication called the distributive law or the distributive property. Given the work that they have done on multiplying by tens and ones on Wednesday, this worksheet should be relatively simple.

Please consider reviewing the Lego poster from earlier this week and checking that your child is remembering those particular number facts more easily.

Here are a few quick grids of the facts that students tend to find the most difficult to recall. Feel free to use them for practise instead. They have fewer calculations, but are the harder ones.

|  | 3 | 4 | 6 | 7 | 8 |
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D18. Distributive Law
Often using mental strategies when multiplying is quicker than using a calculator. Look at the following examples, and work out how the 'distributive law' is being applied.

Distributive Law: Used when breaking up an equation into smaller parts makes it easier.
Examples:

- $19 \times 5=(9 \times 5)+(10 \times 5)=45+50=95$
- $36 \times 9=(30 \times 9)+(6 \times 9)=270+54=324$


## Try these:



What do you think the distributive law does?

How do you know?

Where could you use this?

## BACKWARDS QUESTIONS:

Try to use the distributive law together with what you have learned about extending multiplication facts to solve the following:
$330 \times 4=($
) + (
) $=$ $\qquad$ $+$ $\qquad$ $=$ $\qquad$
$23 \times 50=($
) + (
) $=$ $\qquad$ $+$ $\qquad$ $=$ $\qquad$ $350 \times 70=(\quad)+(\quad)=C_{C}+$
$\qquad$
$\qquad$
$\qquad$

Look up the Distributive Law using a mathematical dictionary and write a definition for it using your own words:

## Teacher Overview

This is a Reasoning lesson. It is designed to determine if students can apply the strategies and processes they have learned for multiplication to calculate multiplication when one of the numbers has two digits. In this lesson, the students work through a two-step process to multiply rather than a single line process. This helps develop the thinking necessary for later algebraic processes, so it is necessary to teach even if it is not as efficient as the single line process.

If you have time online with students, focus on the connection between multiplying by tens and ones. Try other numbers as well. Draw the grids if needed.

